# Stabilisation Policy in a New Keynesian Model with Job Search, Skills Erosion and Growth Effects*
# (Preliminary and incomplete)

Campbell Leith
University of Glasgow

Ioana Moldovan
University of Glasgow

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**Abstract**

This paper develops a sticky-price New Keynesian model where households engage in job search and accumulate both physical and human capital. The accumulation and erosion of human capital differ depending on the time spent in employment and unemployment, with a lower rate of skills erosion and greater positive spillovers for employed versus unemployed workers. Since unemployment affects human capital accumulation it also impacts on endogenous economic growth and hence this is an environment where the costs of unemployment are potentially high and may significantly influence the conduct of monetary policy. In the context of this economy, we explore the nature of optimal monetary policy and the trade-offs it faces in terms of stabilising inflation and output (unemployment). We also analyse the ability of optimal (linear and non-linear) simple policy rules both to deliver a determinate equilibrium and to mimic the fully optimal Ramsey policy.

- *Keywords:* stabilisation policy, nominal inertia, endogenous growth, job search, unemployment, human capital.
- *JEL classification:* E61, E30, E24

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# 1 Introduction

The unemployment figures are among the most widely reported of economic statistics, and the popular debate on economic policy attaches significant weight to trends in that data. Commentaries on interest rate setting decisions by monetary policy makers also appear to attach significant weight to developments in the labour market (see for example, the Inflation Report of the Bank of England or the Monthly Bulletin of the ECB), while full employment is an explicit objective of the US Fed. However, until recently, the benchmark New Keynesian model (as exemplified by Woodford (2003)), which provides the theoretical underpinning to contemporary monetary policy, assumed a Walrasian labour market without any unemployment. This inability of the canonical monetary policy model to comment on the significance of labour market developments for monetary policy has been partially rectified in recent years by introducing unemployment dynamics through the Mortensen-Pissarides (1994) model of job search. However, as we shall see below, while such extensions can help explain labour market fluctuations over the business cycle, they do not imply any significant change in monetary policy practice in that stabilising price inflation remains the primary goal of monetary policy. In this paper, we attempt to enrich the New Keynesian model in a way which captures why policy makers and the public more generally, may care about fluctuations in unemployment. Specifically, we seek to capture the loss of human capital and, subsequently, the lost growth opportunities, that rising unemployment may be thought to imply. Our paper then seeks to assess how such potentially important costs to unemployment influence the conduct of optimal monetary policy.

The literature integrating job search with the New Keynesian model initially focussed on identifying which modelling elements were required to accurately capture labour market dynamics over the business cycle and, in turn, best explain empirical descriptions of the inflation process in the context of both calibrated and estimated models. Here debate has often focussed on the need to incorporate either real wage rigidity (Hall) or staggered nominal wage bargaining (Gertler and Trigari) to overcome the Shimer (2005) puzzle[1] - notable papers in this vein include Sala et al. (2008), Sveen and Weinke (2008), Gali et al. (2011). While the implications of job search for inflation dynamics is considered in Christoffel and Kuester (2008), Christoffel et al. (2009), Krause et al. (2008b) and Sumakama (2011), where some authors have stressed the importance of adopting the right to manage approach to wage determination over the more commonly used assumption of efficient bargaining since the former implies a wage channel, whereby the wage rate feeds directly into the price inflation process, in line with conventional policy making wisdom and some empirical evidence.

More recently, attention has turned to exploring the implications for optimal monetary policy of introducing job search and unemployment into a sticky price New Keynesian economy. Adopting a linear quadratic approach, Ravenna and Walsh (2009) and Blanchard and Gali (2010) demonstrate that, provided the steady-state of the economy is efficient[2], then in the face of productivity shocks monetary policy faces no additional trade-offs and mimicing the flexible price allocation by eliminating inflation remains op-

[1] The puzzle reflects the fact that where the job search model can typically explain wage fluctuations in line with the data, these do not generate sufficient unemployment volatility to match the data. Pissarides (2009) argues that using sticky real or nominal wages to solve the puzzle is not actually consistent with microeconomic evidence on the wages of newly matched hires.

[2] Efficiency in this context requires that a subsidy is applied to eliminate the distortion caused by monopolistic competition, and the weight on the workers' surplus in the Nash product underpinning the wage bargaining following a sucessful match satisfies the Hosios (1990) condition.

timal. A series of papers then relax the assumption that the steady-state of the economy is efficient[3], which implies that policy makers face a trade-off between stabilising inflation and unemployment in the face of technology shocks, although typically the optimal deviations from strict inflation targeting are very small, unless staggered nominal wage bargaining is assumed. However, as Ravenna and Walsh (2011) note, even this is a result of the costs of staggered wage adjustment rather than job search and similar results appear in models without unemployment, but with Calvo contracts in wages as well as prices (see Erceg et al. (2004)). Part of the lack of monetary policy response to fluctuations in unemployment can be explained by the analysis in Ravenna and Walsh (2011), where they argue that monetary policy is simply a poor choice of instrument in offsetting congestion externalities in the labour market at the same time as minimising the costs of nominal inertia, and that the welfare costs of unemployment are actually reasonably high. However, it remains the case that the costs of price dispersion in such models dominate the labour market externalities.

In this paper, we aim to introduce several additional features which accord with our intuition as to why unemployment may matter at an aggregate level (beyond the costs of unemployment experienced by those individuals unlucky enough to suffer from it). Specifically, our model contains households who accumulate both human and physical capital. Human capital accumulation occurs partly exogenously, through a process of compulsory schooling, but also endogenously in that household members who work also undertake on-the-job training. Externalities associated with on-the-job training imply that our model contains endogenous growth effects and that both firms and households will fail to account for these effects when interacting in the labour market. Moreover, workers who enter the state of unemployment will not undertake training and will lose their skills at a faster rate than employed workers, thereby capturing one of the most popular explanations of the costs of unemployment. In order to model this ongoing loss of skills through unemployment tractably, we allow for job sharing within households such that all household members share the same level of human capital. This avoids the need to assume that workers regain their skill levels within one period of regaining employment, as in Laureys (2011) and Rannenberg (2010), or following payment of a fixed retraining cost, as in Esteban-Pretel and Faraglia (2010). It can also help explain features such as the near-hysteretic behaviour of the European labour market where, since human capital must be slowly rebuilt following a spell of unemployment, firms face reduced incentives to post vacancies.

The empirical evidence suggests that US and European labour markets contain significant differences - specifically, Europe suffers from a lower average post-war rate of growth and higher levels of unemployment, the probability of finding a job for an unemployed worker is, cet par, lower in Europe, unemployment benefits are significantly higher compared with the US and persistence in unemployment following shocks is thought to be higher in European economies. Given these differences, we carefully calibrate our economies to capture key features of the European and US economies. We then explore the nature of optimal monetary policy in our sticky price New Keynesian economy augmented with endogenous growth, unemployment, physical capital accumulation, job search and skills loss. Given the wealth of distortions and frictions in such a setup, we follow Ravenna and Walsh (2011) in utilising a raft of fiscal instruments as a means of

[3] This is achieved either by employing the linear-quadratic techniques of Benigno and Woodford (2006) – see, for example, Raissi (2011) and Tang (2006), or by employing higher order solution methods to solve the Ramsey problem in the presence of a distorted steady-state – see, for example, Thomas (2008), Faia (2008, 2009), Ravenna and Walsh (2011) and Sumakama (2011).

quantifying the impact of each inefficiency in isolation and identifying where the key policy trade-offs lie. We also explore the ability of optimal (linear and non-linear) simple monetary policy rules both to deliver a determinate equilibrium and to mimic the fully optimal Ramsey policy.

# 2 The Model

The economy consists of households, intermediate goods and final goods producing firms, and the government. It also features a set of distortions and frictions that create trade-offs for optimal monetary policy acting alone to stabilise the economy in response to exogenous shocks. To better highlight the role of these distortions, we assume the government can implement a set of taxes and subsidies, targetting specific inefficiencies in the economy (more details on the government policy setting are provided below).

## 2.1 Problem of the representative household

Households are large and contain a continuum of members of size 1. We assume that the household's decision making is centralised such that the household takes decisions on behalf of its individual members. For reasons of tractability, we further assume that the household operates a job-sharing scheme within the family, such that any unemployment is shared equally across household members. Such an assumption ensures that, despite the fact that we shall allow human capital accumulation and depreciation to differ during spells of employment and unemployment, each household member still enjoys the same level of human capital, thus negating the need to track the distribution of human capital across household members. Further assuming that the family smoothes consumption and effort levels across household members, by consolidating the household budget constraint and implementing job sharing arrangements, we can consider a representative utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln\left(C_t\right) - \varphi_0 \frac{\left(1-l_t\right)^{1+\varphi}}{1+\varphi} + \left(1-\overline{S}\right) N_t \psi \right]$$

The representative family member spends $\overline{S}$ of her time in compulsory education and $\left(1-\overline{S}\right)$ of her time in the labour market. Of her time in the labour market, $N_t$ is in employment and $U_t$ in unemployment. Equivalently, she spends $\left(1-\overline{S}\right) N_t h_t$ of her time working, $\left(1-\overline{S}\right) N_t \overline{e^N}$ accumulating human capital while in employment, and $\overline{S}\overline{e^S}$ accumulating (or maintaining) human capital while in compulsory education, with the remaining time spent as leisure. This implies the following relationships:

$$1 = l_t + \left(1-\overline{S}\right) N_t (h_t + \overline{e^N}) + \overline{S}\overline{e^S} = U_t + N_t.$$

Finally, $\psi$ captures a utility or status effect related to being employed.

The family's budget constraint, which has been consolidated across family members, is given by

$$\begin{aligned} & P_t C_t + P_t I_t + P_t \phi\left(\frac{I_t}{K_t}\right) K_t + E_t\{Q_{t,t+1} D_{t+1}\} \\ = & \left\{ \begin{array}{c} \left(1-\tau_t\right) W_t h_t H_t \left(1-\overline{S}\right) N_t + P_t \varpi_t \left(1-\overline{S}\right) U_t + D_t + \left(1-\tau_t\right) P_t^K K_t \\ + \left(1-\tau_t\right) P_t \left(\Pi_t - \varkappa_t V_t\right) + P_t \Xi_t M_t - P_t T_t \end{array} \right\} \end{aligned} \quad (1)$$

The household receives after-tax wage income when employed, $(1-\tau_t)W_t h_t H_t\left(1-\overline{S}\right)N_t$, and unemployment benefits when unemployed, $P_t\varpi_t\left(1-\overline{S}\right)U_t$, where $H_t$ is the current level of human capital, $\tau_t$ the income tax rate, and $\varpi_t$ the real value of unemployment benefits. $(1-\tau_t)\,P_t(\Pi_t-\varkappa_t V_t)$ are the household's share of the post-tax profits of the intermediate and final goods firms after paying the (tax deductible) vacancy posting costs required to facilitate a job match, while $P_t\Xi_t M_t$ represent the share of subsidies provided by the government to newly created firms (see Section ... below). $T_t$ are lump-sum taxes paid to the government. The family invests in state contingent assets $D_{t+1}$, where the portfolio includes government debt, $B_{t+1}$. The household also invests $I_t$ in physical capital $K_{t+1}$ and pays $\phi\left(\frac{I_t}{K_t}\right)K_t$ in costs of converting investment into installed capital. The capital is rented to intermediate goods producing firms at the rental price $P_t^K$ and the corresponding revenues are taxed at rate $\tau_t$. With a depreciation rate of $\delta^K$, the family's capital stock evolves according to

$$K_{t+1}=(1-\delta^K)K_t+I_t$$

The family also spends time investing in human capital,

$$\begin{aligned}H_{t+1} &= \left[1-\left(1-\overline{S}\right)N_t\delta^N-\overline{S}\delta^N-\left(1-\overline{S}\right)U_t\delta^U\right]H_t\\ &+\left(1-\overline{S}\right)N_t\left(\overline{A^N}(\overline{e^N}H_t)^{\theta^N}\overline{H}_t^{1-\theta^N}\right)+\overline{S}\left(\overline{A^S}(\overline{e^S}H_t)^{\theta^S}\overline{H}_t^{1-\theta^S}\right)\end{aligned}$$

The rate of depreciation of human capital depends on the proportion of time spent in unemployment versus employment, with $\delta^U>\delta^N$ reflecting the evidence that the unemployed experience a higher erosion of skills relative to the employed (e.g.... CITE EVIDENCE). Family members also accumulate human capital when employed, as they can convert effort $\overline{e^N}$ into human capital according to the function $\overline{A^N}(\overline{e^N}H_t)^{\theta^N}\overline{H}_t^{1-\theta^N}$, where the ability to accumulate capital depends on effort itself, on the existing level of human capital $H_t$, and on a constant productivity term, $\overline{A^N}$.[4] There is also an externality in that human capital accumulation is easier if the average level of human capital in others, $\overline{H}_t$, is greater. If $\theta^N=1$, that externality is removed. A similar accumulation of human capital occurs when in schooling, but there is no human capital accumulation when unemployed. We treat the extent of compulsory schooling $\left(\overline{S}\right)$ and the effort employed in gaining skills $\left(\overline{e^N}\text{ and }\overline{e^S}\right)$ as being exogenous and constant – introducing compulsory schooling enables us to match the growth data without assuming that all human capital accumulation takes place through on-the-job training, while a constant effort is more in line with the evidence on cyclical variations in job-related training programs [Evidence...] [5] Given this specification, human capital accumulation and economic growth are tightly linked to employment dynamics and the degree of skills erosion.

The household's utility maximising choices are given by the Euler equation,

$$1=\beta E_t\left(\frac{u_{c,t+1}}{u_{c,t}}\pi_{t+1}^{-1}\right)R_t$$

[4]Although this productivity term may depend on government investment in training for employed workers, it is unlikely that monetary policy have a similar effect. We hence assume it constant across the business cycle.

[5]When allowing for endogenous effort $e_t^N$, the household attempts to exploit this channel to a very high degree, not consistent with observations on the cyclical variation in on-the-job training, and leads to unstable paths under an optimal monetary setting. We consider the policy implications of endogenous labour force participation and schooling in Leith et al. (2015b).

the first order condition for physical capital, $K_{t+1}$,

$$u_{c,t}\left(1+\phi'\left(\frac{I_t}{K_t}\right)\right)=\beta E_t u_{c,t+1}\left[\begin{array}{c}(1-\tau_{t+1})\,p^K_{t+1}-\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)+\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\\ +\left(1+\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\left(1-\delta^K\right)\end{array}\right] \quad (2)$$

and that for human capital, $H_{t+1}$,

$$\lambda_{3t}=\beta E_t\lambda_{1t+1}\left[(1-\tau_{t+1})\,w_{t+1}h_{t+1}\left(1-\overline{S}\right)N_{t+1}\right]+$$

$$+\beta E_t\lambda_{3t+1}\left[\begin{array}{c}\left(1-\left(1-\overline{S}\right)N_{t+1}\delta^N-\overline{S}\delta^N-\left(1-\overline{S}\right)U_{t+1}\delta^U\right)\\ +\left(1-\overline{S}\right)N_{t+1}\left(\theta^N\frac{\overline{A^N}(e^NH_{t+1})^{\theta^N}\overline{H}^{1-\theta^N}_{t+1}}{H_{t+1}}\right)+\overline{S}\left(\theta^S\frac{\overline{A^S}(e^SH_{t+1})^{\theta^S}\overline{H}^{1-\theta^S}_{t+1}}{H_{t+1}}\right)\end{array}\right] \quad (3)$$

where $R_t=(E_tQ_{t,t+1})^{-1}$ is the gross nominal interest rate on one-period riskless bonds, $\pi_t=P_t/P_{t-1}$ is the gross rate of inflation, $p^K_t=\frac{P^K_t}{P_t}$ the real rental price of capital, and $\lambda_{3t}$ represents the shadow value of human capital.[6] In the absence of adjustment costs, the first order condition for physical capital (2) becomes the usual Euler equation for capital,

$$u_{c,t}=\beta E_t u_{c,t+1}\left[(1-\tau_{t+1})\,p^K_{t+1}+1-\delta^K\right]$$

while the valuation of human capital in equation (3), taken from the point of view of individual households, will help us highlight externalities arising from the labour market.

## 2.2 Intermediate Goods Firms

There are two production sectors: the intermediate goods sector and the final goods sector. The intermediate goods sector consists of a continuum of firms, indexed by $i$ and of measure $(1-\overline{S})N_t$, which employ labour and hire capital to produce differentiated intermediate goods in imperfectly competitive markets. Final goods firms (also a continuum but of measure 1) then use a CES aggregate of the intermediate goods to obtain a homogenous good, which they subsequently convert into differentiated goods and sell at profit maximising prices. Firms in the final goods sector are also subject to nominal inertia in the form of Calvo (1983)-type contracts.

Denoting with $Y^{I,i}_t$ the output of intermediate firm $i$, the overall CES aggregate of intermediate goods (across all final goods producers) is given by

$$Y^I_t=\left[\int_0^{(1-\overline{S})N_t}\left(Y^{I,i}_t\right)^{\frac{\epsilon^I-1}{\epsilon^I}}di\right]^{\frac{\epsilon^I}{\epsilon^I-1}}$$

with an associated price index $P^I_t=\left[\int_0^{(1-\overline{S})N_t}\left(P^{I,i}_t\right)^{1-\epsilon^I}di\right]^{\frac{1}{1-\epsilon^I}}$ and a corresponding demand curve $Y^{I,i}_t=\left(\frac{P^{I,i}_t}{P^I_t}\right)^{-\epsilon^I}Y^I_t$, for each intermediate good $i$. The CES specification includes a love of variety effect, as in Benassy (2006), which is associated with an externality arising from the fact that new intermediate goods firms (and hence new varieties)

[6] See Appendix A for details of the derivations.

are created through individual matches in the labour market that ignore the aggregate effect on output. In a symmetric equilibrium, we have $Y_t^I = \left[\left(1-\overline{S}\right)N_t\right]^{\nu+1} Y_t^{I,i}$ where $\nu = 1/\left(\epsilon^I - 1\right)$ captures the love of variety.

The production function for intermediate goods is given by

$$Y_t^{I,i} = A_t \left(K_t^i\right)^{\alpha} \left(h_t H_t\right)^{1-\alpha} \tag{4}$$

where $A_t$ is a stationary productivity term, common across firms, and following the AR(1) process $\ln A_t = (1-\rho_A)\ln A + \rho_A \ln A_{t-1} + \varepsilon_t^A$, with $\varepsilon_t^A \sim iid\left(0,\sigma^A\right)$. This implies that the profits of intermediate goods firm $i$ are, in real terms,

$$\begin{aligned}\Pi_t^{I,i} &= \left(1-\tau_t^I\right) x_t A_t \left(K_t^i\right)^{\alpha} \left(h_t H_t\right)^{1-\alpha} - w_t h_t H_t - \left(1-\upsilon_t\right) p_t^K K_t^i \\ &= \left(1-\tau_t^I\right) \left[\left(1-\overline{S}\right)N_t\right]^{\nu} \frac{P_t^I}{P_t} A_t \left(\frac{K_t}{\left(1-\overline{S}\right)N_t}\right)^{\alpha} \left(h_t H_t\right)^{1-\alpha} - w_t h_t H_t - \left(1-\upsilon_t\right) p_t^K \frac{K_t}{\left(1-\overline{S}\right)N_t}\end{aligned} \tag{5}$$

where $x_t \equiv \frac{P_t^{I,i}}{P_t}$ is the real price of intermediate good $i$. The second line expresses individual profits in terms of aggregate variables, where we have used the fact that, by symmetry, $\int_0^{(1-\overline{S})N_t} K_t^i di = \left[\left(1-\overline{S}\right)N_t\right] K_t^i = K_t$ and $P_t^I = \left[\left(1-\overline{S}\right)N_t\right]^{-\nu} P_t^{I,i}$, implying $x_t = \left[\left(1-\overline{S}\right)N_t\right]^{\nu} p_t^I$, with $p_t^I \equiv \frac{P_t^I}{P_t}$ as the relative price of the composite of intermediate goods. Finally, $\tau_t^I$ is a tax on intermediate goods firms' revenues and $\upsilon_t$ a subsidy to the costs of hiring capital.

While hours are determined alongside wages in the bargaining process (see below), the firm's optimal choice of capital, which maximises profits subject to the technology and demand constraints, is given by:

$$\left(1-\tau_t^I\right) x_t mpk_t = \left(\frac{\epsilon^I}{\epsilon^I - 1}\right)\left(1-\upsilon_t\right) p_t^K$$

where $mpk_t = \alpha \frac{Y_t^{I,i}}{K_t^i}$ is the marginal product of capital. This is the usual condition under imperfect competition, where the firm hires capital up to the point where its after-tax marginal value product is at a markup over the subsidised cost. Using this relationship, together with the optimal choice of hours from equation (10) and the production function (4), yields the following implicit pricing condition:

$$\left(1-\tau_t^I\right) x_t = \alpha^{-\alpha} \left(1-\alpha\right)^{\alpha-1} A_t^{-1} \left(\frac{\epsilon^I}{\epsilon^I - 1}\left(1-\upsilon_t\right) p_t^K\right)^{\alpha} \left(\frac{mrs_t/H_t}{1-\tau_t}\right)^{1-\alpha}.$$

### 2.3 The Labour Market

Given the job sharing arrangements in place within the representative family, it is not the case that an individual is being matched to a job. Instead the family are trying to find a new job to divide amongst its members. We follow Christoffel et al. (2009) in formulating the matching side of the economy. The matching technology is

$$M_t = \sigma_m \left(\left(1-\overline{S}\right)U_t\right)^{\xi} \left(V_t\right)^{1-\xi}$$

where $U_t$ is the rate of unemployment and $V_t$ are the available vacancies. Defining the labour market tightness as $\theta_t = \frac{V_t}{(1-\overline{S})U_t}$, the probability of filling a vacancy is $z_t = M_t/V_t = \sigma_m (\theta_t)^{-\xi}$, while the probability of the family converting a unit of time from a state of unemployment to a state of employment is $s_t = M_t/\left(\left(1-\overline{S}\right)U_t\right) = \sigma_m (\theta_t)^{1-\xi}$.

We assume that existing matches are destroyed at an exogenous rate $\vartheta$ every period, while newly formed matches start working with a one period delay. This implies the following evolution of employment

$$\left(1-\overline{S}\right)N_t = (1-\vartheta)\left(1-\overline{S}\right)N_{t-1} + M_{t-1}$$

The value to the family of possessing a job is given by

$$V_t^E = (1-\tau_t)w_t h_t H_t - \frac{u_{l,t}(h_t + \overline{e^N})}{u_{c,t}} + \frac{\psi}{u_{c,t}} + E_t q_{t,t+1}\left[(1-\vartheta)V_{t+1}^E + \vartheta V_{t+1}^U\right]$$

and includes four terms: the after-tax real wage income, the cost of leisure forgone when in employment (expressed in consumption units)[7], the utility benefit of employment (also expressed in consumption units), and finally the continuation value (as a weighted average of the expected future values of employment and unemployment, with weights given by the separation rate $\vartheta$). $q_{t,t+1}$ is the stochastic discount factor for real payoffs, $q_{t,t+1} = \beta\frac{u_{c,t+1}}{u_{c,t}}$.

The payoffs from not possessing a job are given by the unemployment benefits $\varpi_t$ (in real terms) and a similar continuation value (where the weights are now given by the probability of finding a job, $s_t$):

$$V_t^U = \varpi_t + E_t q_{t,t+1}\left[s_t V_{t+1}^E + (1-s_t)V_{t+1}^U\right]$$

The net value of a job to the family can thus be written as

$$\left(V_t^E - V_t^U\right) = (1-\tau_t)w_t h_t H_t - \varpi_t - \frac{u_{l,t}(h_t + \overline{e^N})}{u_{c,t}} + \frac{\psi}{u_{c,t}} + E_t q_{t,t+1}\left[(1-\vartheta-s_t)\left(V_{t+1}^E - V_{t+1}^U\right)\right] \quad (6)$$

The value of a match to an intermediate firm, $J_t$, is given by its current after-tax real profits and the value of the discounted expected future profits, should the match survive into the next period. In addition, we assume the government supports the creation of new jobs via a lump-sum subsidy $\Xi_t$ to firms entering the market, hence the expression for $J_t$ is:

$$J_t = (1-\tau_t)\Pi_t^{I,i} + \Xi_t + E_t q_{t,t+1}\left[(1-\vartheta)J_{t+1}\right] \quad (7)$$

Potential intermediate goods firms (or the households that own them) must pay a cost $(1-\tau_t)\varkappa_t$ to post a vacancy.[8] Since there are no impediments to posting vacancies, they will be posted until they equal the discounted expected value of any match that could emerge

$$(1-\tau_t)\varkappa_t = E_t\left(q_{t,t+1} z_t J_{t+1}\right)$$

Together with the expression for $J_t$, the job creation condition, written in terms of the

[7] The forgone leisure associated with one job includes the time devoted to work, $h_t$, and the time spent in training to accumulate human capital, $\overline{e^N}$.

[8] We are assuming that the vacancy posting costs are tax deductible.

market tightness parameter $\theta_t$, is:

$$\frac{(1-\tau_t)\varkappa_t}{\sigma_m\theta_t^{-\xi}} = E_t q_{t,t+1}\left[(1-\tau_{t+1})\Pi_{t+1}^{I,i} + \Xi_{t+1} + (1-\vartheta)\frac{(1-\tau_{t+1})\varkappa_{t+1}}{\sigma_m\theta_{t+1}^{-\xi}}\right] \tag{8}$$

Note that unemployment benefits $\varpi_t$, the vacancy costs $\varkappa_t$, as well as the subsidy $\Xi_t$, must be rising over time to ensure stationarity and we assume they grow in line with the general level of human capital in the economy.

### 2.3.1 Family(Worker) - Firm Bargaining

Families and firms that have made a match bargain over real wages and hours worked, in order to maximise the Nash product of their respective surpluses, $\left(V_t^E - V_t^U\right)^\eta (J_t)^{1-\eta}$. The first order condition for the real wage is

$$\eta J_t = (1-\eta)\left(V_t^E - V_t^U\right) \tag{9}$$

and for hours worked it is

$$mrs_t = (1-\tau_t)\left(1-\tau_t^I\right)x_t mpl_t \tag{10}$$

where $mrs_t \equiv \frac{u_{l,t}}{u_{c,t}}$ is the marginal rate of substitution between consumption and leisure and $mpl_t \equiv (1-\alpha)\frac{Y_t^{I,i}}{h_t}$ is the marginal product of labour of intermediate goods firms.

### 2.3.2 Further Manipulations

Using the bargained wage condition (9) and the definitions of profits $\Pi_t^{I,i}$ and of $\left(V_t^E - V_t^U\right)$ and $J_t$ in equations (5) - (7), we obtain the real wage bill as a weighted average of the returns from a match to the firm and to the household, reflecting the nature of wage determination in the labour search framework (see Appendix A for details)

$$\begin{aligned} w_t h_t H_t &= \eta\left[\left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_t^I\right)x_t Y_t^{I,i} + \frac{\Xi_t}{1-\tau_t} + \theta_t\varkappa_t\right] \\ &\quad + (1-\eta)\left[\frac{\varpi_t + mrs_t\left(h_t + \overline{e^N}\right) - \psi/u_{c,t}}{1-\tau_t}\right] \end{aligned}$$

Then, together with the definition of profits, the job creation condition (8) can be written as follows

$$\frac{(1-\tau_t)\varkappa_t}{\sigma_m\theta_t^{-\xi}} = \beta E_t\frac{u_{c,t+1}}{u_{c,t}}\left\{\begin{array}{c} (1-\eta)\left[\begin{array}{c} (1-\tau_{t+1})\left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_{t+1}^I\right)x_{t+1}Y_{t+1}^{I,i} \\ -mrs_{t+1}\left(h_{t+1}+\overline{e^N}\right) + \psi/u_{c,t+1} - \varpi_{t+1} + \Xi_{t+1} \end{array}\right] \\ +(1-\vartheta-\eta\sigma_m\theta_{t+1}^{1-\xi})\frac{(1-\tau_{t+1})\varkappa_{t+1}}{\sigma_m\theta_{t+1}^{-\xi}} \end{array}\right\}$$

This expression is closer in form to that delivered by the social planner's problem and will allow us to better highlight some of the distortions arising from the labour market.

## 2.4 Final Goods Sector

Firm $i$ in the final goods sector converts the homogeneous intermediate good (obtained as the aggregate of all intermediate goods) into a final differentiated product, $Y_t^i$. It does so using a linear technology, $Y_t^i = Y_t^I(i)$, where $Y_t^I(i)$ is the amount of the homogenous intermediate good used as input in production. The total nominal cost of producing final good $i$ is given by $P_t^I Y_t^I(i)$, implying a nominal marginal cost $MC_t = P_t^I$ (and an associated real marginal cost $mc_t = p_t^I$), common across firms.

Firms supply their products to households and the government for the purposes of consumption and investment. Each of these sectors aggregates these differentiated goods into the same CES-type basket, such that their cumulative demand for each differentiated final good is given by

$$Y_t^i = \left(\frac{P_t^i}{P_t}\right)^{-\epsilon} Y_t \tag{11}$$

where $Y_t = C_t + I_t + \phi\left(\frac{I_t}{K_t}\right) K_t + \varkappa_t V_t + G_t$ and $P_t \equiv \left(\int_0^1 \left(P_t^i\right)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the associated final goods price index.

Final goods producers are also subject to the constraints of Calvo (1983)-contracts such that, with fixed probability $(1-\omega)$ in each period, a firm can reset its price and with probability $\omega$ it retains the price of the previous period. When a firm can set the price, it does so in order to maximise the present discounted value of after-tax profits, $E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s}\left(1-\tau_{t+s}\right)\left[\left(\left(1-\tau_t^{\dagger}\right) P_t^i - MC_t\right) Y_t^i\right]$, and subject to the demand for its own good (11) and the constraint that all demand be satisfied at the chosen price. Profits are discounted by the $s$-step ahead stochastic discount factor $Q_{t,t+s}$ and by the probability of not being able to set prices in future periods. $\tau_t^{\dagger}$ is a revenues tax/subsidy which we use as part of our set of tax policy instruments that can render the decentralised equilibrium efficient. Optimally, the relative price satisfies the following relationship:

$$\frac{\widetilde{P}_t}{P_t} = \left(1-\tau_t^{\dagger}\right)^{-1}\left(\frac{\epsilon}{\epsilon-1}\right) \frac{E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left(1-\tau_{t+s}\right) u_{c,t+s} p_{t+s}^I \left(\frac{P_t}{P_{t+s}}\right)^{-\epsilon} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left(1-\tau_{t+s}\right) u_{c,t+s} \left(\frac{P_t}{P_{t+s}}\right)^{1-\epsilon} Y_{t+s}}$$

With $\omega$ of firms keeping last period's price and $(1-\omega)$ of firms setting a new price, the final goods price index evolves according to:

$$P_t^{1-\epsilon} = (1-\omega)\, \widetilde{P}_t^{(1-\epsilon)} + \omega P_{t-1}^{1-\epsilon}$$

## 2.5 The Government

The government purchases goods for public consumption $G_t$ and pays unemployment benefits $\varpi_t$. It levies a general income tax $\tau_t$ on households' income (including wage income, income from capital rentals and net profits from ownership of firms). It also raises taxes $\tau_t^I$ and $\tau_t^{\dagger}$ on the revenues of intermediate goods firms and final goods firms, respectively, and provides a subsidy $\upsilon_t$ for capital hiring and a subsidy $\Xi_t$ to newly created firms. The resulting net government budget deficit is financed via lump-sum taxation on households $T_t$ (where, in case of a surplus, $T_t$ represents a subsidy).

$$G_t+\varpi_t\left(1-\overline{S}\right)U_t+\upsilon_t p_t^K K_t+\Xi_t M_t=\left\{\begin{array}{c}\tau_t\left[w_t h_t H_t\left(1-\overline{S}\right)N_t+\left(\Pi_t-\varkappa_t V_t\right)+p_t^K K_t\right]\\+\tau_t^I p_t^I\left(S_t^p Y_t\right)+\tau_t^\dagger Y_t+T_t\end{array}\right\} \tag{12}$$

We consider monetary policy set optimally with commitment, but also explore alternative policy settings in the form of simple Taylor-type rules (see Section 7). In assessing the role of the various distortions in our economy and their implications for the trade-offs faced by monetary policy, we will assign specific roles to the set of tax instruments $\left\{\tau_t,\tau_t^I,\tau_t^\dagger,\upsilon_t,\Xi_t\right\}$, as detailed in Section 4. Finally, government expenditures are assumed to grow in line with the level of human capital, $G_t=g_t H_t$, and we allow for exogenous cyclical variations in $g_t$, specified as: $\ln g_t=(1-\rho_G)\ln\overline{g}+\rho_G\ln g_{t-1}+\varepsilon_t^G$, with $\varepsilon_t^G\sim iid\left(0,\sigma^G\right)$.

### 2.6 Aggregation and Stationarity

Appendix B provides the details of aggregation, which implies that aggregate output is given by

$$S_t^p Y_t=\left[(1-\overline{S})N_t\right]^\nu A_t\left(K_t\right)^\alpha\left(h_t H_t(1-\overline{S})N_t\right)^{1-\alpha} \tag{13}$$

which is the standard constraint, adjusted for the existence of price dispersion, $S_t^p\equiv\int_0^1\left(\frac{P_t^i}{P_t}\right)^{-\epsilon}di$, and for variations in the number of intermediate goods varieties (induced by the presence of unemployment/compulsory schooling). The price dispersion term follows an AR(1) process (see **?**, Chapter 6):

$$S_t^p=(1-\omega)\left(\frac{\widetilde{P}_t}{P_t}\right)^{-\epsilon}+\omega\pi_t^\epsilon S_{t-1}^p. \tag{14}$$

Combining the household's budget constraint (1) with the government budget constraint (12) and the definition of profits ($\Pi_t=\left(1-\tau_t^\dagger-\tau_t^I p_t^I S_t^p\right)Y_t-w_t h_t H_t\left(1-\overline{S}\right)N_t-(1-\upsilon_t)\,p_t^K K_t$) and given that, in equilibrium, net financial assets are zero in the absence of government debt, the aggregare resource constraint is:

$$C_t+I_t+\phi\left(\frac{I_t}{K_t}\right)K_t+\varkappa_t V_t+G_t=Y_t. \tag{15}$$

Equilibrium conditions are then rendered stationary by expressing aggregate non-stationary variables relative to the current level of human capital. Appendix B lists the entire set of equilibrium conditions in both non-stationary and stationary forms. Here we only include those equations to which we make specific reference in the rest of the paper. Defining $x_t\equiv X_t/H_t$, where $X_t=\{K_t,Y_t,C_t,I_t\}$, and $\widetilde{x}_t\equiv X_t/H_t$, where $X_t=\{\Xi_t,mrs_t,mpl_t,mpk_t,u_{c,t}\}$, and $\varpi\equiv\frac{\varpi_t}{H_t}$, $\varkappa\equiv\frac{\varkappa_t}{H_t}$, $\widetilde{\lambda}_{3t}\equiv\lambda_{3t}H_t$, and $\gamma_t\equiv H_{t+1}/H_t$, we then have:

The first order condition for physical capital:

$$\widetilde{u}_{c,t}\left(1+\phi'\left(\frac{i_t}{k_t}\right)\right)\gamma_t=\beta E_t\widetilde{u}_{c,t+1}\left[\begin{array}{c}(1-\tau_{t+1})\,p_{t+1}^K-\phi\left(\frac{i_{t+1}}{k_{t+1}}\right)+\phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)\frac{i_{t+1}}{k_{t+1}}\\+\left(1+\phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)\right)\left(1-\delta^K\right)\end{array}\right] \tag{16}$$

The value of a unit of human capital:

$$\begin{aligned}\widetilde{\lambda}_{3t}\gamma_t &= \beta E_t \widetilde{u}_{c,t+1}\left[\left(1-\tau_{t+1}\right) w_{t+1} h_{t+1}\left(1-\overline{S}\right) N_{t+1}\right] \\ &+ \beta E_t \widetilde{\lambda}_{3t+1}\left[\begin{array}{c}\left(1-\left(1-\overline{S}\right) N_{t+1}\delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right) U_{t+1}\delta^U\right) \\ +\left(1-\overline{S}\right) N_{t+1}\left(\theta^N \overline{A^N}\left(\overline{e^N}\right)^{\theta^N}\right) + \overline{S}\left(\theta^S \overline{A^S}\left(\overline{e^S}\right)^{\theta^S}\right)\end{array}\right]\end{aligned} \tag{17}$$

Hours decision:

$$\widetilde{mrs}_t = \left(1-\tau_t\right)\left(1-\tau_t^I\right) x_t \widetilde{mpl}_t \tag{18}$$

Intermediate goods firms' choice of capital:

$$\left(1-\tau_t^I\right) x_t \widetilde{mpk}_t = \left(\frac{\epsilon^I}{\epsilon^I - 1}\right)\left(1-\upsilon_t\right) p_t^K \tag{19}$$

Job creation dynamics:

$$\frac{\left(1-\tau_t\right)\varkappa}{\sigma_m \theta_t^{-\xi}} = \beta E_t \frac{\widetilde{u}_{c,t+1}}{\widetilde{u}_{c,t}}\left\{\begin{array}{c}\left(1-\eta\right)\left[\begin{array}{c}\left(1-\tau_{t+1}\right)\left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_{t+1}^I\right) x_{t+1} y_{t+1}^{I,i} \\ -\widetilde{mrs}_{t+1}\left(h_{t+1}+\overline{e^N}\right)+\psi/\widetilde{u}_{c,t+1}-\varpi+\widetilde{\Xi}_{t+1}\end{array}\right] \\ +\left(1-\vartheta-\eta\sigma_m\theta_{t+1}^{1-\xi}\right)\frac{\left(1-\tau_{t+1}\right)\varkappa}{\sigma_m\theta_{t+1}^{-\xi}}\end{array}\right\} \tag{20}$$

## 3 A Constrained Social Planner's Problem

We consider a social planner's problem, where the planner is constrained by the workings of the labour market (see, for example, Tomas (2008), Arseneau and Chugh (2009), and Faia (2009)), and which delivers a constrained efficient allocation that allows for a measure of the 'best' employment level. Specifically, the social planners chooses real allocations to maximise the representative household's utility subject to the usual constraints (including the aggregate resource contraint, the production technology, the evolution of physical and human capital), as well as the constraints pertaining to the labour market (the evolution of employment and the matching technology). Consistently with the assumptions made in the decentralised equilibrium, we also impose that effort for the accumulation of human capital $e^N$ is constant, while the vacancy posting costs $\varkappa_t$ and government spending $G_t$ grow in line with human capital.[9] We present below the optimal choices that arise from this problem, already in stationary form, with full details included in Appendix C.

The choice of hours worked $h_t^*$ is such that marginal rate of substitution between consumption and leisure equals an aggregate measure of the marginal product of labour:

$$\widetilde{mrs}_t^* = \left[(1-\overline{S})N_t^*\right]^\nu \ \widetilde{mpl}_t^* \tag{21}$$

where $\widetilde{mpl}_t^* \equiv (1-\alpha)\frac{y_t^{i*}}{h_t^*}$ is the marginal product of labour of the individual firm.

Similarly, the first order condition for physical capital equates the cost of additional

[9]In the absence of utility benefits from public goods consumption, the social planner would optimally set $G_t = 0$. However, in order to be able to compare the outcome in the decentralized economy with the social planner's allocation, we assume $G_t = g_t H_t^*$ but abstract from cyclical fluctuations in $g_t$ such that $g_t = \overline{g}$ at all times.

investment (inclusive of the marginal adjustment costs of capital conversion) to the expected future benefits, evaluated from the view of the aggregate economy:

$$\widetilde{u}^*_{c,t}\left(1+\phi'\left(\frac{i^*_t}{k^*_t}\right)\right)\gamma^*_t = \beta E_t \widetilde{u}^*_{c,t+1}\left[\begin{array}{c} \left[(1-\overline{S})N^*_t\right]^{\nu}\widetilde{mpk}^*_t - \phi\left(\frac{i^*_{t+1}}{k^*_{t+1}}\right) + \phi'\left(\frac{i^*_{t+1}}{k^*_{t+1}}\right)\frac{i^*_{t+1}}{k^*_{t+1}} \\ +\left(1+\phi'\left(\frac{i^*_{t+1}}{k^*_{t+1}}\right)\right)\left(1-\delta^K\right) \end{array}\right] \tag{22}$$

where $\widetilde{mpk}^*_t \equiv \alpha\frac{y^{i*}_t}{k^{i*}_t}$ is the marginal product of capital of the individual firm and the first term on the right hand side gives the corresponding measure at an aggregate level.

The first order condition for human capital is

$$\widetilde{\lambda}^*_{3t}\gamma^*_t = \beta E_t \widetilde{u}^*_{c,t+1}\left[(1-\alpha)\,y^*_{t+1} - \varkappa V_{t+1} - \overline{g}\right] + \beta E_t \widetilde{\lambda}^*_{3t+1}\gamma^*_{t+1} \tag{23}$$

and captures the value of human capital from the point of view of society as a whole. It includes the expected positive effect on future aggregate output, net of increased government expenditures and vacancy-posting costs, as well as the value of a higher rate of economic growth.

Finally, the social planner's optimal vacancy posting/job creation condition is given by the following expression:

$$\frac{\varkappa}{\sigma_m\left(\theta^*_t\right)^{-\xi}} = \beta E_t \frac{\widetilde{u}^*_{c,t+1}}{\widetilde{u}^*_{c,t}}\left\{\begin{array}{c} (1-\xi)\left[\begin{array}{c} (\nu+1-\alpha)\frac{y^*_{t+1}}{(1-\overline{S})N^*_{t+1}} - \widetilde{mrs}^*_{t+1}\left(h^*_{t+1}+\overline{e^N}\right) + \frac{\psi}{\widetilde{u}^*_{c,t+1}} \\ -\frac{\widetilde{\lambda}^*_{3t+1}}{\widetilde{u}^*_{c,t+1}}\left(\delta^N - \delta^U - \overline{A^N}\left(\overline{e^N}\right)^{\theta^N}\right) \end{array}\right] \\ +\left[1-\vartheta-\xi\sigma_m\left(\theta^*_{t+1}\right)^{1-\xi}\right]\frac{\varkappa}{\sigma_m\left(\theta^*_{t+1}\right)^{-\xi}} \end{array}\right\} \tag{24}$$

## 4 Market Frictions and the Role of Policy

Our model economy is characterised by a number of frictions and distortions: (*i*) nominal inertia in price setting; (*ii*) monopolistic competition in both sectors of production; (*iii*) a set of externalities, arising from the atomistic nature of the labour market, which affect vacancy posting and job creation dynamics, with further implications for the economy and particularly for the accumulation of human capital and growth. Such an array of distortions creates trade-offs for the monetary policy maker using a single policy instrument. To better assess the significance of these distortions and their impact on monetary policy setting, we follow the 'tax approach' of Ravenna and Walsh (2011) and assign to our tax instruments specific roles that target different distortions. This is such that, used together, all policy instruments allow the policy maker to reproduce the (constrained) social planner's allocation.

Firstly, let monetary policy be solely concerned with ensuring price stability, in which case price dispersion is eliminated, $S^p_t = 1$, and final goods prices are set at a constant markup over marginal cost. Defining $\mu_t \equiv \frac{P_t}{P^I_t}$ as the ratio of the final goods prices over the nominal marginal cost, price stability in our model gives:

$$\mu_t = \overline{\mu} = \frac{\epsilon/(\epsilon-1)}{1-\tau^{\dagger}} \tag{25}$$

To offset the effects of monopolistic competition in the final goods sector, we set the subsidy $\tau^{\dagger}$ to a constant

$$1-\tau^{\dagger}=\frac{\epsilon}{\epsilon-1} \tag{26}$$

which further implies $\mu_t=\overline{\mu}=1$.

An efficient choice of hours requires $\widetilde{mrs}_t=\left[(1-\overline{S})N_t\right]^{\nu}\ \widetilde{mpl}_t$, as in the social planner's condition (21). The bargaining outcome for hours in (18) satisfies this condition if

$$(1-\tau_t)\left(\frac{1-\tau_t^I}{\mu_t}\right)=1 \tag{27}$$

Comparison of the Euler equations for physical capital (16) and (22) suggests that the choice of capital is optimal if the after-tax rental price of capital equals an aggregate measure of the marginal product of capital, $(1-\tau_t)\,p_t^K=\left[\left(1-\overline{S}\right)N_t\right]^{\nu}\widetilde{mpk}_t$. An expression for $p_t^K$, obtained from the firm's choice of capital in (19), together with the fact that the relative price of intermediate goods can be written as $x_t=\left[\left(1-\overline{S}\right)N_t\right]^{\nu}\frac{1}{\mu_t}$, indicates that efficiency is achieved if

$$(1-\tau_t)\left(\frac{1-\tau_t^I}{\mu_t}\right)\left(\frac{\epsilon^I-1}{\epsilon^I}\frac{1}{1-\upsilon_t}\right)=1 \tag{28}$$

Hence, if hours are chosen efficiently (as per the condition in (27)), then the capital rental subsidy $\upsilon_t$ in (28) can be set to offset the effects of the monopolistic competition externality in the intermediate goods sector and thus ensure an efficient choice of capital,

$$1-\upsilon=\frac{\epsilon^I-1}{\epsilon^I} \tag{29}$$

Finally, we compare the job creation condition in the decentralised equilibrium (20) with that of the social planner (24), in order to highlight those externalities arising from the labour market and affecting the posting of vacancies and job creation dynamics. The output related term in the social planner's condition, $(\nu+1-\alpha)\frac{y_{t+1}^*}{(1-\overline{S})N_{t+1}^*}$, captures the effect of an additional job on aggregate output and consists of a positive 'aggregate effect', inclusive of love-of-variety effects, equal to $(\nu+1)\frac{y_{t+1}^*}{(1-\overline{S})N_{t+1}^*}$ and a negative 'capital-intensity effect' of $(-\alpha)\frac{y_{t+1}^*}{(1-\overline{S})N_{t+1}^*}$, reflecting the fact that an increase in the number of jobs/firms reduces the level of capital available for each individual firm, $\frac{k_t}{(1-\overline{S})N_t^*}$, thus reducing production. The net effect is however positive. The equivalent term in the decentralised equilibrium condition can be re-written as,

$$\begin{aligned}
&(1-\tau_{t+1})\left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_t^I\right)x_{t+1}y_{t+1}^{I,i}\\
=\ &(1-\tau_{t+1})\left(\frac{1-\tau_{t+1}^I}{\mu_{t+1}}\right)\left[(\nu+1-\alpha)-\nu+\frac{\alpha}{\epsilon^I}\right]\frac{y_{t+1}}{(1-\overline{S})N_{t+1}}
\end{aligned}$$

The efficiency condition for hours worked (27) makes the product of the tax instruments $(1-\tau_{t+1})\left(\frac{1-\tau_{t+1}^I}{\mu_{t+1}}\right)=1$. Then, the remaining term reveals what we shall call the *'output externality'*, whereby firms/workers fail to fully account for the effect of an additional job on economy-wide output by an amount equal to[10]

$$\widetilde{\Xi}^{Y}_{t+1} = \left(\nu - \frac{\alpha}{\epsilon^{I}}\right) \frac{y_{t+1}}{(1-\overline{S})N_{t+1}} \tag{30}$$

The second externality is associated with the unemployment benefits $\varpi$, which raise the net value of a job to the household and hence increase the bargained wage, with subsequent negative effects on job creation. We let

$$\widetilde{\Xi}^{UB}_{t+1} = \varpi. \tag{31}$$

A third externality, to which we refer to as the *'human capital externality'*, arises from the fact that firms/workers do not internalise the effects of job creation on the accumulation of human capital, as does the social planner. This externality is captured by the term

$$\widetilde{\Xi}^{H}_{t+1} = -\frac{\widetilde{\lambda}_{3t+1}}{\widetilde{u}_{c,t+1}} \left(\delta^{N} - \delta^{U} - \overline{A^{N}}\left(\overline{e^{N}}\right)^{\theta^{N}}\right) \tag{32}$$

which appears in the social planner's condition (24) but is missing from the decentralised equilibrium condition (20). With $\delta^N < \delta^U$, $\widetilde{\Xi}^{H}_{t+1} > 0$, hence this externality makes the labour market too tight and unemployment inefficiently high. An accurate measure of $\widetilde{\Xi}^{H}_{t+1}$ requires an optimal valuation of human capital, $\widetilde{\lambda}_{3t+1}$, from the perspective of the economy as a whole, as given by social planner in equation (23). We assume the government is able to undertake such an evaluation.[11]

To compensate for these three externalities, the government sets the lump-sum subsidy $\widetilde{\Xi}_t$, provided to newly created firms, to:

$$\widetilde{\Xi}_{t+1} = \widetilde{\Xi}^{Y}_{t+1} + \widetilde{\Xi}^{UB}_{t+1} + \widetilde{\Xi}^{H}_{t+1} \tag{33}$$

With this subsidy in place, the only remaining inefficiency in job creation is due to the congestion externality, arising from deviations from the Hosios (1990) condition for efficiency, which requires the firms bargaining power, $(1-\eta)$, to equal the elasticity of the matching function with respect to vacancies, $(1-\xi)$. If $\eta < \xi$, then intermediate goods firms have strong incentives to post vacancies and unemployment is inefficiently low. To correct this inefficiency, the government can use the income tax $\tau_t$ to ensure the job creation condition in the decentralised economy matches that of the social planner. This in turn implies that the tax on intermediate firms' revenues $\tau^I_t$ must be such that the efficiency condition for the choice of hours (27) is satisfied. If the Hosios condition holds ($\eta = \xi$) and given the subsidy $\widetilde{\Xi}_{t+1}$ and all other policy instruments are in place, then job creation and hours worked are efficient with $\tau_t = \tau^I_t = 0$.

Through the judicious setting of policy, as given in the efficiency conditions (25)-(33), the decentralised economy is able to achieve the social planner's allocation.

[10]This term is positive in the presence of love-of-variety effects $\left(i.e.\ \nu = 1/\left(\epsilon^I - 1\right)\right)$, but negative when $\nu = 0$.

[11]We note that the valuation of human capital $\widetilde{\lambda}_{3t}$ in the decentralised equilibrium, in eqn. (17), is from the point of view of the household and hence different from that of the social planner. This discrepancy highlights further externalies in our economy, which we can however ignore, as $\widetilde{\lambda}_{3t}$ does not affect the rest of the economy, when effort $e^N_t$ is exogenous.

## 5 Calibration

We undertake two calibrations of the model designed to replicate the U.S. and European economies. Table 1 contains a list of the model parameters and their calibrated values. For some parameters (denoted with '$*$' in the table), we impose values based on existing estimates in the literature and some of these (denoted with '$**$') are common for the two economies considered. The rest of the parameters are calibrated to match a set of steady state relationships and data moments from the two economies over the period 1980:Q1-2008:Q4 for the U.S. and 1980:Q1- 2005:Q4 for Europe.

The household discount factor and rate of depreciation of physical capital are standard. The quarterly discount factor is set equal to 0.99, implying an annual rate of time preference of 4%, as is standard in the literature. In combination with the other parameters, this implies a steady-state annual real interest rate of 7% and 6% for the U.S. and Europe, respectively. This is obviously far higher than the risk free rate observed in the data and reflects the well-known risk-free rate puzzle of Weil (1989) whereby representative agent models typically imply a very high risk-free rate of interest (for a discussion see Ireland (2004)). An alternative approach would have been to reduce the household's rate of time preference in $\beta$ in order to achieve the observed risk-free real rate (see, for example, Smets and Wouters, 2007). However, this would greatly inflate the discounted costs of shocks in our model. The rate of depreciation of human capital during employment reflects the estimates of DeJong and Ingram (2001). While the rate of depreciation of human capital when unemployed implies a dip in salary of 15% following a one year spell of unemployment consistent with the range of estimates discussed in Rannenberg(2009). The parameter in the adjustment cost function, $\mu = 1.78$, is calibrated in line with the estimates of this parameter obtained from regressions based on the q-model of investment (see, for example, Eberly (1997) and the extensive discussion of the empirical evidence in Barlevy (2004))

Further imposed parameters include the labour supply elasticity which comes from the Bayesian estimation of Smets and Wouters (2005), the estimates for price stickiness reflects the estimation in Leith and Malley (2005). The matching elasticity with respect to unemployment is set to 0.72 for the U.S. based on the estimates of Shimer (2005) and to 0.6 for Europe following Christoffel et al (2009). The level of unemployment benefits/insurance was chosen to achieve a U.S. replacement ratio of 15% (this reflects the estimates in Pallage et al (2008) for Ohio which the authors argue to be representative of the U.S. as a whole) and the far higher ratio of 65% for Europe from Christoffel et al (2009). While the relatively modest human capital accumulation externalities reflected in $\theta^N = 0.9$ come from the evidence cited in de la Croix and Doepke (2003). The mark-up in the final and intermediate goods sectors are set to 9.1% in both economies, implying an overall mark-up of 19% in line with typical calibrations of New Keynesian models (see, for example, Leith and Malley, 2005).

The remaining parameters $\{A^N, A^S, \alpha, \varphi_0, \eta, \vartheta, A, \psi\}$ were chosen to achieve a set of steady-state relationships detailed in Table 1. Specifically, that 1/3 of time spent in employment was spent working, (Erceg et al, 2000) and 0.2 of that time was spent on human capital augmenting on the job training (see Kim and Lee, 2007), while 0.3 is the proportion of time spent in school (see the evidence presented in Angelopoulous et al, 2008). The calibration also ensures the model generated the labour share values, average post-1980 growth rates and observed unemployment rates across the two economies. Estimates from the OECD study on Human Capital (Lui, 2011) suggest ratios of Human Capital to GDP of around 9.5, with a comparable ratio for physical capital of 2.2. While the estimates for the Euro area are incomplete, there are a range of estimates across European economies, such that assuming similar ratios in Europe does not seem unreasonable. The disutility costs of unemployment, $\psi$, were calibrated such that the they offset the leisure gain due to being unemployed. This is slightly above the calibrated value in Gali (2010) based on time devoted to job search when unemployed, but as noted by Gali is below the values that would be consistent with the very large costs of unemployment found in the happiness literature (Frey, MIT book). The fact that reservation wages are often estimated to be below the value of unemployment benefits also suggest that there are significant utility costs to unemployment (see Bloemen, 1997).

The calibration also recreates a key stylised fact that the probability of exiting unemployment and finding a job is significantly higher in the U.S. than Europe (while the calibrated separation rates are only modestly higher in the U.S., consistent with empirical estimates - see Hobijn and Sahin, 2007). Moreover, workers' bargaining power is calibrated to be around 0.33 in Europe relative to the lower figure of 0.27 in the US, which captures the greater degree of unionisation in Europe. The shares of government consumption to GDP are imposed following the evidence in Gali (1994). The calibration implies a reasonable composition of GDP in the two economies, while the implied discounted value of a match is around 24% of annual salary in the U.S., and 26% in Europe, where European firms pay lower vacancy posting costs. The various calibrated technology parameters are similar across the two economies, with marginally higher productivity, cet. par. in the U.S. and more efficient accumulation of human capital while working in the U.S.

**Table 1: Parameters and their calibrated values for the US and Europe**

| Parameter | US | Europe | Description |
|---|---|---|---|
| *Preferences* | | | |
| $\beta$ ∗∗ | 0.99 | 0.99 | Household discount factor |
| $\varphi$ ∗ | 2.88 | 2 | Labour supply elasticity. |
| $\varphi_0$ | 90.6 | 39.8 | Weight on labour supply in utility. Targets h = 1/3. |
| $\psi$ | 1.95 | 2.01 | Relative Disutility Cost of Unemployment |
| *Human capital accumulation* | | | |
| $\delta^N$ ∗∗ | 0.005 | 0.005 | Rate of depreciation of human capital when in employment. |
| $\delta^U$ ∗∗ | 0.0395 | 0.0395 | Rate of depreciation of human capital when unemployed. |
| $\theta^N \left(= \theta^S\right)$ ∗∗ | 0.9 | 0.9 | Elasticity of human capital accumulation to effort when employed (and in schooling). |
| $A^N$ | 0.050 | 0.047 | Efficiency of human capital accumulation when employed. |
| $A^S$ | 0.096 | 0.086 | Efficiency of human capital accumulation when in schooling. |
| *Physical capital accumulation and intermediate goods production* | | | |
| $\delta^K$ ∗∗ | 0.015 | 0.015 | Rate of depreciation of physical capital. |
| $\varsigma$ ∗∗ | 1.78 | 1.78 | Investment adjustment cost parameter. |
| $\varsigma_0$ ∗∗ | 0.2 | 0.2 | Investment adjustment cost scaling parameter. |
| $\alpha$ | 0.34 | 0.32 | Weight on physical capital in goods production function. |
| $A$ | 0.127 | 0.125 | Technology parameter, intermediate goods production. |
| $\epsilon^I/(\epsilon^I - 1)$ ∗∗ | 9.1% | 9.1% | Mark-up for intermediate goods firms. |
| *Labour market bargaining* | | | |
| $\eta$ | 0.27 | 0.33 | Workers' bargaining power. |
| $\xi$ ∗ | 0.72 | 0.6 | Elasticity of matches w.r.t. unemployment. |
| $\sigma_m$ | 0.83 | 0.3 | Efficiency of matching function. |
| $\varkappa$ | 0.048 | 0.015 | Vacancy posting costs |
| $\vartheta$ | 0.0558 | 0.0294 | Separation rate |
| $\varpi$ | $0.15 \times wh$ | $0.65 \times wh$ | Unemployment benefits. Targets replacement ratio. |
| *Final goods production* | | | |
| $\epsilon/(\epsilon - 1)$ ∗∗ | 9.1% | 9.1% | Mark-up for final goods firms. |
| $\omega$ ∗ | 0.58 | 0.78 | Probability of no price change within a quarter. |
| *Exogenous shocks* | | | |
| $\rho_A$ | | | Autocorrelation of technology shock. |
| $\rho_G$ | | | Autocorrelation of government spending shock. |
| $\sigma_A$ | | | Standard deviation of technology shock. |
| $\sigma_G$ | | | Standard deviation of government spending shock. |

*Notes:* The table reports calibrated values for the US and Europe. '∗' denotes an imposed value and '∗∗' a value that is common for the US and Europe. Other parameters are calibrated to data from 1980:Q1-2008:Q4 for the US and 1980:Q1-2005:Q4 for Europe. And inflation is zero ($\pi = 1$).

|  | US | Europe | Description |
|---|---|---|---|
| $\overline{S}$ | 0.3 | 0.3 | Proportion of time spent in school. |
| $h$ | 1/3 | 1/3 | Hours worked when employed. |
| $\overline{e^N}$ | $0.2 \times h$ | $0.2 \times h$ | Human capital effort while in employment. |
| $\overline{e^S}$ | 1/3 | 1/3 | Human capital effort while in school. |
| $U$ | 0.063 | 0.0893 | Unemployment rate. |
| $s$ | 0.83 | 0.3 | Quarterly probability of finding a job. |
| $V/\left(\left(1-\overline{S}\right)U\right)$ | 1 | 1 | Vacancy to unemployment rate (normalization) |
| $\gamma^4$ | 2.88% | 2.06% | Annual growth rate |
| $H/(4y)$ | 9.5 | 9.5 | Human capital to output ratio |
| $k/(4y)$ | 2.2 | 2.2 | Physical capital to output ratio |
| $g/y$ | 0.16 | 0.20 | Government spending to output ratio |
| $wh\left(1-\overline{S}\right)N/y$ | 0.54 | 0.6 | Labour income share. |

Table 1: Data elements matched in the calibration.

|  | US | Europe | Description |
|---|---|---|---|
| $i/y$ | 0.19 | 0.17 | Investment/Output Ratio |
| $c/y$ | 0.57 | 0.59 | Consumption/Output Ratio |
| $\varkappa V/y$ | 0.077 | 0.037 | Vacancy Posting Costs/GDP |
| $r$ | 7.10% | 6.22% | Real Interest Rate (annualised) |
| $(v^E - v^U)/(4wh)$ | 0.24 | 0.26 | Value of a match as proportion of salary. |

Table 2: Additional implied steady-state values

# 6 Optimal Monetary Policy

## 6.1 The long-run

| | Decentr. Econ. | Hosios | mc_I | mc_F | Hosios +mc_I +mc_F |
|---|---|---|---|---|---|
| $\gamma^4$ | 2.88% (data) | 2.48% | 2.89% | 2.89% | 2.51% |
| $c$ | 0.0149 | 0.0161 | 0.0155 | 0.016 | 0.0178 |
| $y$ | 0.0263 | 0.0258 | 0.0277 | 0.0285 | 0.0294 |
| $c/y$ | 0.57 | 0.62 | 0.56 | 0.56 | 0.61 |
| $i/y$ | 0.19 | 0.19 | 0.21 | 0.21 | 0.23 |
| $k/(4y)$ | 2.2 (data) | 2.27 | 2.39 | 2.39 | 2.69 |
| $N$ | 93.7% (data) | 90.13% | 93.77% | 93.83% | 90.4% |
| $U$ | 6.3% (data) | 9.87% | 6.23% | 6.17% | 9.6% |
| $V/\left(\left(1-\overline{S}\right)U\right)$ | 1 | 0.175 | 1.042 | 1.08 | 0.196 |
| $h$ | 0.333 (data) | 0.337 | 0.335 | 0.344 | 0.35 |
| $leisure$ | 0.638 | 0.646 | 0.636 | 0.629 | 0.637 |
| $welfare$ | -267.82 | -271.12 | -264.64 | -264.02 | -263.89 |

Table 3: Steady-state results, US calibration. The columns correspond to: (1) fully decentralized economy, (2) Hosios condition holds, (3) market power of intermediate goods firms offset by cost subsidy, (4) market power of final goods firms offset by revenues subsidy, (5) cumulative effects of (2)-(4).

| | Decentr. Econ. | Hosios | mc_I | mc_F | Hosios +mc_I +mc_F |
|---|---|---|---|---|---|
| $\gamma^4$ | 2.04% (data) | 1.64% | 2.07% | 2.10% | 1.75% |
| $c$ | 0.0154 | 0.0159 | 0.0159 | 0.0166 | 0.0177 |
| $y$ | 0.0263 | 0.0263 | 0.0275 | 0.0285 | 0.03 |
| $c/y$ | 0.59 | 0.61 | 0.58 | 0.58 | 0.59 |
| $i/y$ | 0.18 | 0.17 | 0.19 | 0.19 | 0.21 |
| $k/(4y)$ | 2.2 (data) | 2.28 | 2.39 | 2.39 | 2.68 |
| $N$ | 91.07% (data) | 87.43% | 91.35% | 91.65% | 88.46% |
| $U$ | 8.93% (data) | 12.57% | 8.65% | 8.35% | 11.54% |
| $V/\left(\left(1-\overline{S}\right)U\right)$ | 1 | 0.384 | 1.089 | 1.202 | 0.489 |
| $h$ | 0.333 (data) | 0.344 | 0.334 | 0.345 | 0.356 |
| $leisure$ | 0.645 | 0.649 | 0.644 | 0.636 | 0.638 |
| $welfare$ | -298.68 | -308.27 | -295.01 | -293.48 | -298.78 |

Table 4: Steady-state results, Europe calibration. The columns correspond to: (1) fully decentralized economy, (2) Hosios condition holds, (3) market power of intermediate goods firms offset by cost subsidy, (4) market power of final goods firms offset by revenues subsidy, (5) cumulative effects of (2)-(4).

| | Hosios +mc_I +mc_F | +Y_ext | +UB | +H_ext | Social Planner |
|---|---|---|---|---|---|
| $\gamma^4$ | 2.51% | 2.54% | 2.54% | 2.78% | 2.79% |
| $c$ | 0.0178 | 0.0178 | 0.0178 | 0.0172 | 0.0171 |
| $y$ | 0.0294 | 0.0294 | 0.0294 | 0.0296 | 0.0296 |
| $c/y$ | 0.61 | 0.61 | 0.60 | 0.58 | 0.58 |
| $i/y$ | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 |
| $k/(4y)$ | 2.69 | 2.69 | 2.69 | 2.64 | 2.64 |
| $N$ | 90.4% | 90.65% | 90.68% | 92.79% | 92.92% |
| $U$ | 9.6% | 9.35% | 9.32% | 7.21% | 7.08% |
| $V/\left(\left(1-\overline{S}\right)U\right)$ | 0.196 | 0.217 | 0.219 | 0.596 | 0.641 |
| $h$ | 0.35 | 0.349 | 0.349 | 0.345 | 0.345 |
| $leisure$ | 0.637 | 0.636 | 0.636 | 0.632 | 0.632 |
| $\Xi$ | 0 | 0.0029 | 0.0033 | 0.0533 | 0.0592 |
| $\Xi/c$ | 0 | 0.163 | 0.183 | 3.11 | 3.46 |
| $welfare$ | -263.89 | -263.33 | -263.27 | -260.27 | -260.25 |

Table 5: Steady-state results, US calibration - cont'd. Column (1): Hosios condition holds and monopoly power in both sectors offset via subsidies. In columns (2)-(4), the subsidy 'capxi' is then used to separately account for: aggregate output effects, unemployment benefits, human capital externalities. (See Section X for details). With all instruments used appropriately, the social planner's allocation in column (5) is obtained.

In Tables 3-6 we examine the implications of removing the various distortions in our model. We do this progressively, by considering various market power distortions in Table 3 for the US and Table 4 for Europe, before considering the remaining labour market distortions in Tables 5 and 6, respectively. We begin with Table 3 where the first column details the calibrated steady-state of our decentralised equilibrium. The next three columns remove various forms of market power - namely the bias in bargaining towards firms implied by the deviation from the Hosios condition and monopolistic competition in the intermediate and final goods sectors, respectively. The final column removes all three forms of market power simultaneously. Here we can see that reducing the power of firms in the Nash bargain to the levels implied by the Hosios condition leads to a significant increase in unemployment from 6.3% to European levels of 9.87% and an associated rise in labour market tightness ($\theta$ falls). Associated with the higher unemployment, we observe a fall in the growth rate from the 2.88% recorded in the data (and imposed in the calibration) to 2.48%, which leads to a fall in steady-state welfare, despite slightly higher levels of consumption and leisure. In contrast, reducing the monopoly power, in either the intermediate or final goods sectors, implies a modest fall in unemployment and an increase in growth and welfare. Simultaneously removing these 'market-power' distortions is dominated by the effects of imposing the Hosios condition in terms of reduced long-run growth rates, higher unemployment rates and a tighter labour market. However, welfare is marginally improved due to increased consumption levels. A similar pattern is found for Europe in Table 4, except that the combined elimination of the three forms of market power is slightly welfare reducing in this case.

Next, Table 5 begins with the decentralised equilibrium after the removal of the three sources of 'market power' analysed in Table 3 and considers the marginal effect of removing three externalities which firms/workers do not take account of when bargaining over wages and deciding to post vacancies. The first of these is the fact that each additional job created increases aggregate output, primarily via increases in the variety of intermediate

| | Hosios +mc_I +mc_F | +Y_ext | +UB | +H_ext | Social Planner |
|---|---|---|---|---|---|
| $\gamma^4$ | 1.75% | 1.84% | 2.06% | 2.31% | 2.38% |
| $c$ | 0.0177 | 0.0176 | 0.0173 | 0.0167 | 0.0166 |
| $y$ | 0.03 | 0.0298 | 0.0298 | 0.0301 | 0.0303 |
| $c/y$ | 0.59 | 0.59 | 0.58 | 0.56 | 0.55 |
| $i/y$ | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 |
| $k/(4y)$ | 2.68 | 2.66 | 2.61 | 2.56 | 2.55 |
| $N$ | 88.46% | 89.23% | 91.27% | 93.55% | 94.15% |
| $U$ | 11.54% | 10.77% | 8.73% | 6.45% | 5.85% |
| $V/\left(\left(1-\overline{S}\right)U\right)$ | 0.489 | 0.596 | 1.065 | 2.417 | 3.134 |
| $h$ | 0.356 | 0.353 | 0.348 | 0.345 | 0.345 |
| $leisure$ | 0.638 | 0.638 | 0.635 | 0.631 | 0.629 |
| $\Xi$ | 0 | 0.0031 | 0.0161 | 0.0516 | 0.0699 |
| $\Xi/c$ | 0 | 0.174 | 0.932 | 3.08 | 4.22 |
| $welfare$ | -298.78 | -296.66 | -291.63 | -287.84 | -287.54 |

Table 6: Steady-state results, Europe calibration - cont'd. Column (1): Hosios condition holds and monopoly power in both sectors offset via subsidies. In columns (2)-(4), the subsidy 'capxi' is then used to separately account for: aggregate output effects, unemployment benefits, human capital externalities. (See Section X for details). With all instruments used appropriately, the social planner's allocation in column (5) is obtained.

goods, which is akin to an improvement in productivity in that sector. This increases the annualised growth rate by around 0.03% and reduces the unemployment rate by 0.25%. Similar effects on growth and unemployment arise when considering the disincentives created by unemployment benefits, as indicated in column 3 of the table. While the fourth column subsidises the creation of a job so as to ensure firms/workers internalise the impact on human capital accumulation of the creation of an additional job. In the U.S., the marginal impact on the annualised growth rate is 0.27% and unemployment falls by 2.39%. Combining all three labour market distortions, together with the removal of 'market power' in the intermediate and final goods markets, as well as imposing the Hosios condition, allows us to recreate the constrained social planner's allocation, which is detailed in the final column. Interestingly, while individual distortions appear to have very large effects (particularly the failure to impose the Hosios condition and the failure of firms/workers to take account of the impact of their actions on aggregate human capital accumulation), the net impact of all the distortions taken together is relatively modest, due to partially offsetting effects. Relative to the benchmark economy, we observe a marginal increase in unemployment of 0.78% and a small reduction in the growth rate of 0.09%. Nevertheless, the removal of the numerous distortions does enable the U.S. economy to accumulate more capital and produce more with an associated increase in welfare, due to higher long-run consumption.

Finally, Table 6 undertakes the same exercise for Europe by removing the effects of the remaining labour market distortions. For the European economy the impact of these labour market distortions is significantly greater. Taking account of the benefits of job creation on the expanded variety of intermediate goods raises the European growth rate by 0.09%, with an associated fall in unemployment of 0.77% (this is about three times the equivalent effect for the U.S. economy). Removing the disincentives created by the high European unemployment benefits has even larger effects compared to the U.S. counterpart, with a marginal impact on growth of 0.31% and a fall in the unemployment rate

| | US calibration | | | Europe calibration | | | |
|---|---|---|---|---|---|---|---|
| | $\varpi = 0.0033$ | $\varpi^{high} = 0.0161$ | $\varpi^{opt} = 0.012$ | $\varpi = 0.0161$ | $\varpi^{low} = 0.0033$ | $\varpi = 0$ | $\varpi^{opt} = -0$ |
| $\varpi/(wh)$ | 0.15 | 0.69 | 0.53 | 0.65 | 0.14 | 0 | $-0.2$ |
| $\gamma^4$ | 2.88% | 2.72% | 2.79% | 2.04% | 2.29% | 2.33% | 2.38% |
| $c$ | 0.0149 | 0.0156 | 0.0153 | 0.0154 | 0.0149 | 0.0148 | 0.0147 |
| $y$ | 0.0263 | 0.0259 | 0.026 | 0.0263 | 0.0266 | 0.0267 | 0.0268 |
| $c/y$ | 0.57 | 0.60 | 0.59 | 0.59 | 0.56 | 0.56 | 0.55 |
| $i/y$ | 0.19 | 0.19 | 0.19 | 0.18 | 0.18 | 0.18 | 0.18 |
| $k/(4y)$ | 2.2 | 2.23 | 2.22 | 2.2 | 2.15 | 2.15 | 2.14 |
| $N$ | 93.7% | 92.23% | 92.85% | 91.07% | 93.43% | 93.76% | 94.14% |
| $U$ | 6.3% | 7.77% | 7.15% | 8.93% | 6.57% | 6.24% | 5.86% |
| $\frac{V}{(1-\overline{S})U}$ | 1 | 0.455 | 0.617 | 1 | 2.3 | 2.64 | 3.12 |
| $h$ | 0.333 | 0.332 | 0.332 | 0.333 | 0.330 | 0.330 | 0.331 |
| $leisure$ | 0.638 | 0.642 | 0.641 | 0.645 | 0.640 | 0.639 | 0.638 |
| $welfare$ | -267.82 | -267.35 | -267.03 | -298.68 | -294.69 | -294.45 | -294.35 |

Table 7: Table Caption

of 2.81%. While if workers and firms were to internalise the human capital externalities associated with job creation, there would be a massive fall in unemployment of 5.1% and increase in the annualised growth rate of 0.56%. Combining these effects leads us to the social planner's allocation described in the final column. In contrast to the case of the U.S., where the net effects of the various distortions had a negligible impact on steady-state growth rates and unemployment, the European situation is very different: the social planner would achieve a growth rate of 2.48%, far higher than the calibrated value of 2.04%, whereas the unemployment rate would fall from 8.93% to 5.85%, which is below that of the U.S.

These differences across the US and Europe reflect the importance of labour market distortions in the calibrated European economy.

### 6.1.1 Unemployment Benefits

A significant difference across the two economies lies in the calibrated size of the benefit-replacement ratios which were only 0.15 in the U.S., but 0.65 in Europe. In this subsection, we explore the implications of unemployment benefits in each economy moving closer to those in the other economy. Table X recreates Table 3 for the U.S. but where the unemployment benefit has been increased to European levels. In this second best world such a policy change is actually welfare improving in all cases other than when it is associated with the simultaneous imposition of the Hosios condition. Similarly, Table Y recreates Table 4 for Europe after reducing unemployment benefits to U.S. levels. In this case, in the decentralised equilibrium, regardless of the removal of the various types of market power, such unemployment benefit is welfare improving. Accordingly, it appears to be the case that the U.S. would benefit from higher unemployment benefits and Europe from lower. This begs the question as to what the optimal level of unemployment benefits is. A grid search over alternative values indicates that the optimal benefit-replacement ratio in the U.S. and Europe are 0.53 and -0.2, respectively.

### 6.2 Short-run dynamics

## 7 Monetary Policy Rules

### 7.1 Determinacy

We first assume a simple standard Taylor-type rule, where the nominal interest rates responds to inflation and output and features a degree of inertia

$$\ln(R_t/R) = \psi_\pi \ln(\pi_t/\pi) + \psi_y \ln(y_t/y) + \psi_R \ln(R_{t-1}/R)$$

We then consider an alternative rule, where the policy responds to changes in unemployment, instead of output.

$$\ln(R_t/R) = \psi_\pi \ln(\pi_t/\pi) + \psi_U \ln(U_t/U) + \psi_R \ln(R_{t-1}/R)$$

### 7.2 Impulse Responses under Policy Rules

We now outline the response of the model to two types of shocks: a technology shock and a government spending shock, with alternative variants of our policy rules. In respect of the technology shock the....

### 7.3 Welfare Comparisons

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# A Analytical Details

## A.1 Households Utility Maximisation

We solve the following Lagrangian of the utility maximisation problem:

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \beta^t \{u\left(C_t, l_t, N_t\right) - \\
&\quad -\lambda_{1t} \left[ \begin{array}{c} C_t + I_t + \phi\left(\frac{I_t}{K_t}\right) K_t + E_t\{Q_{t,t+1} D_{t+1}\}/P_t - (1-\tau_t) w_t h_t H_t \left(1-\overline{S}\right) N_t \\ -\varpi_t \left(1-\overline{S}\right) U_t - (1-\tau_t) p_t^K K_t - D_t/P_t - (1-\tau_t)\left(\Pi_t - \varkappa_t V_t\right) - \Xi_t M_t + T_t \end{array} \right] \\
&\quad -\lambda_{2t}[K_{t+1} - (1-\delta^K) K_t - I_t] \\
&\quad -\lambda_{3t} \left[ \begin{array}{c} H_{t+1} - (1 - \left(1-\overline{S}\right) N_t \delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right) U_t \delta^U) H_t \\ -\left(1-\overline{S}\right) N_t \left( \overline{A^N} (e^N H_t)^{\theta^N} \overline{H}_t^{1-\theta^N} \right) - \overline{S} \left( \overline{A^S} (e^S H_t)^{\theta^S} \overline{H}_t^{1-\theta^S} \right) \end{array} \right] \}
\end{aligned}
$$

The first order conditions are:
Consumption, $C_t$ :

$$u_{c,t} = \lambda_{1t} \tag{34}$$

Financial assets, $D_{t+1}$ :

$$1 = \beta E_t \left( \frac{u_{c,t+1}}{u_{c,t}} \pi_{t+1}^{-1} \right) R_t$$

where $R_t = \left(E_t Q_{t,t+1}\right)^{-1}$ is the one-period gross return on riskless bonds and $\pi_{t+1}$ is the gross rate of inflation between periods $(t)$ and $(t+1)$.

Investment, $I_t$ :

$$\lambda_{1t} \left(1 + \phi' \left(\frac{I_t}{K_t}\right)\right) = \lambda_{2t} \tag{35}$$

Physical capital, $K_{t+1}$ :

$$\lambda_{2t} = \beta E_t \lambda_{1t+1} \left[ \left(1-\tau_{t+1}^K\right) p_{t+1}^K - \phi\left(\frac{I_{t+1}}{K_{t+1}}\right) + \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} \right] + \beta E_t \lambda_{2t+1} \left(1-\delta^K\right) \tag{36}$$

Human Capital, $H_{t+1}$ :

$$
\begin{aligned}
\lambda_{3t} &= \beta E_t \lambda_{1t+1} \left[ \left(1-\tau_{t+1}\right) w_{t+1} h_{t+1} \left(1-\overline{S}\right) N_{t+1} \right] + \\
&\quad +\beta E_t \lambda_{3t+1} \left[ \begin{array}{c} (1 - \left(1-\overline{S}\right) N_{t+1} \delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right) U_{t+1} \delta^U) \\ + \left(1-\overline{S}\right) N_{t+1} \left( \theta^N \frac{\overline{A^N} (\overline{e^N} H_{t+1})^{\theta^N} \overline{H}_{t+1}^{1-\theta^N}}{H_{t+1}} \right) + \overline{S} \left( \theta^S \frac{\overline{A^S} (\overline{e^S} H_{t+1})^{\theta^S} \overline{H}_{t+1}^{1-\theta^S}}{H_{t+1}} \right) \end{array} \right]
\end{aligned}
$$

Combining the first order conditions for investment and capital, (35) and (36), with the first order condition for consumption (34), we obtain the following condition

$$u_{c,t}\left(1+\phi'\left(\frac{I_t}{K_t}\right)\right)=\beta E_t u_{c,t+1}\left[\begin{array}{c}(1-\tau_{t+1})\,p^K_{t+1}-\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)+\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\\ +\left(1+\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\left(1-\delta^K\right)\end{array}\right]$$

which, in the absence of adjustment costs ($\phi(\cdot)=\phi'(\cdot)=0$), becomes the usual Euler equation for capital,

$$u_{c,t}=\beta E_t u_{c,t+1}\left[(1-\tau_{t+1})\,p^K_{t+1}+1-\delta^K\right].$$

**Functional forms:**
Utility is given by $u(C,l,N)=\ln(C)-\varphi_0\frac{(1-l)^{1+\varphi}}{1+\varphi}+\left(1-\overline{S}\right)N\psi$, hence the first derivatives are:

$$u_c(\cdot)=C^{-1}\qquad \textit{and}\qquad u_l(\cdot)=\varphi_0(1-l)^{\varphi}$$

while the capital adjustment costs function is

$$\phi\left(\frac{I}{K}\right)=\frac{\varsigma_0}{\varsigma}\left(\frac{I}{K}\right)^{\varsigma},\qquad \varsigma_0\geq 0,\quad \varsigma>1$$

implying a marginal cost,

$$\phi'\left(\frac{I}{K}\right)=\varsigma_0\left(\frac{I}{K}\right)^{\varsigma-1}.$$

## A.2 Intermediate Goods Firms - Pricing Decision

Intermediate goods firm $i$'s profits, in real terms, are given by:

$$\Pi^{I,i}_t=\left(1-\tau^I_t\right)x_tA_t\left(K^i_t\right)^{\alpha}(h_tH_t)^{1-\alpha}-w_th_tH_t-(1-\upsilon_t)\,p^K_tK^i_t$$

where $x_t\equiv\frac{P^{I,i}_t}{P_t}$ is the real price of intermediate good $i$. The firm also faces a downward sloping demand, $Y^{I,i}_t=\left(\frac{P^{I,i}_t}{P^I_t}\right)^{-\epsilon^I}Y^I_t$. The firm's optimal choice of capital, given the technology and demand constraints, satisfies the following relationship:

$$\left(1-\tau^I_t\right)x_tmpk_t=\left(\frac{\epsilon^I}{\epsilon^I-1}\right)(1-\upsilon_t)\,p^K_t$$

where $mpk_t=\alpha\frac{Y^{I,i}_t}{K^i_t}$ is the marginal product of capital of the intermediate firm. At the same time, hours worked are determined in the bargaining process by condition (10)

$$mrs_t=(1-\tau_t)\left(1-\tau^I_t\right)x_tmpl_t$$

From these two conditions, we obtain the following expressions for capital and hours worked:

$$K^i_t=\frac{\alpha\left(1-\tau^I_t\right)x_tY^{I,i}_t}{\left(\frac{\epsilon^I}{\epsilon^I-1}\right)(1-\upsilon_t)\,p^K_t}$$

and

$$h_t = \frac{(1-\alpha)\left(1-\tau_t^I\right) x_t Y_t^{I,i}}{\left(\frac{1}{1-\tau_t}\right) mrs_t}$$

which, upon substitution into the production function, yield an implicit pricing condition:

$$\left(1-\tau_t^I\right) x_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} A_t^{-1}\left(\frac{\epsilon^I}{\epsilon^I-1}(1-\upsilon_t)\, p_t^K\right)^{\alpha}\left(\frac{mrs_t/H_t}{1-\tau_t}\right)^{1-\alpha}$$

or, equivalently,

$$\left(1-\tau_t^I\right) p_t^I = \left(\left(1-\overline{S}\right) N_t\right)^{-\nu}\left[\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} A_t^{-1}\left(\frac{\epsilon^I}{\epsilon^I-1}(1-\upsilon_t)\, p_t^K\right)^{\alpha}\left(\frac{mrs_t/H_t}{1-\tau_t}\right)^{1-\alpha}\right]$$

## A.3 Bargained Wage and Job Creation Condition

**The bargained wage:** We obtain an expression for the bargained wage, as a weighted average of the returns from the match to the firm and to the household. First, accounting for its choice of capital, the intermediate firm's profits can be written as:

$$\Pi_t^{I,i} = \left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_t^I\right) x_t Y_t^{I,i} - w_t h_t H_t \tag{37}$$

Second, substituting the expressions for $\left(V_t^E - V_t^U\right)$ and $J_t$, from equations (6) and (7), into the wage bargaining condition (9) yields the following:

$$\begin{aligned} w_t h_t H_t &= \eta\left[\left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_t^I\right) x_t Y_t^{I,i} + \frac{\Xi_t}{1-\tau_t} + \frac{f_t^F}{1-\tau_t}\right] \\ &\quad + (1-\eta)\left[\frac{\varpi_t + mrs_t\left(h_t + \overline{e^N}\right) - \psi/u_{c,t}}{1-\tau_t} - \frac{f_t^W}{1-\tau_t}\right] \end{aligned}$$

where $f_t^F$ and $f_t^W$ are the expected net present values from the match to the firm and to the household. The worker is compensated for a fraction $\eta$ of the return from the match to the firm (i.e. the firm's current revenues net of the cost of capital, plus the government subsidy $\Xi_t$ and the expected future net present value from employment, $f_t^F$) and for a fraction $(1-\eta)$ of the foregone value of being unemployed (i.e. the foregone unemployment benefits, the consumption value of foregone leisure net of the utility gain associated with employment status, and the household's forgone expected future net present value from unemployment, $-f_t^W$).

Using the vacancy-posting condition and the bargained wage relationship, $f_t^F$ and $f_t^W$ can be re-written as

$$\begin{aligned} f_t^F &\equiv E_t q_{t,t+1}\left[(1-\vartheta)J_{t+1}\right] = (1-\vartheta)(1-\tau_t)\frac{\varkappa_t}{\sigma_m\theta_t^{-\xi}} \\ f_t^W &\equiv E_t q_{t,t+1}\left[(1-\vartheta-s_t)\left(V_{t+1}^E - V_{t+1}^U\right)\right] \\ &= E_t q_{t,t+1}\left[(1-\vartheta-s_t)\left(\frac{\eta}{1-\eta}\right) J_{t+1}\right] = (1-\vartheta-s_t)\left(\frac{\eta}{1-\eta}\right)(1-\tau_t)\frac{\varkappa_t}{\sigma_m\theta_t^{-\xi}} \end{aligned}$$

which implies that real wage income depends on contemporaneous variables only:

$$
\begin{aligned}
w_t h_t H_t &= \eta\left[\left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_t^I\right)x_t Y_t^{I,i}+\frac{\Xi_t}{1-\tau_t}+\theta_t\varkappa_t\right] \\
&\quad +(1-\eta)\left[\frac{\varpi_t+mrs_t\left(h_t+\overline{e^N}\right)-\psi/u_{c,t}}{1-\tau_t}\right]
\end{aligned}
\tag{38}
$$

Dividing through by $h_t H_t$ gives the expression for the real wage, which is similar to that obtained by Trigari (2006) and reflects the nature of bargaining in the context of labour market search.

**The job creation condition:**

Substituting for the real wages and profits (from equations (38) and (37) above), the job creation condition (8) becomes:

$$
\frac{(1-\tau_t)\varkappa_t}{\sigma_m\theta_t^{-\xi}}=\beta E_t\frac{u_{c,t+1}}{u_{c,t}}\left\{\begin{array}{c}(1-\eta)\left[\begin{array}{c}(1-\tau_{t+1})\left(1-\alpha\frac{\epsilon^I-1}{\epsilon^I}\right)\left(1-\tau_{t+1}^I\right)x_{t+1}Y_{t+1}^{I,i} \\ -mrs_{t+1}\left(h_{t+1}+\overline{e^N}\right)+\psi/u_{c,t+1}-\varpi_{t+1}+\Xi_{t+1}\end{array}\right] \\ +(1-\vartheta-\eta\sigma_m\theta_{t+1}^{1-\xi})\frac{(1-\tau_{t+1})\varkappa_{t+1}}{\sigma_m\theta_{t+1}^{-\xi}}\end{array}\right\}
$$

# B Equilibrium Conditions

## B.1 Aggregation

**Aggregate output:** To obtain a measure of aggregate output, we consider the firm's production function, $Y_t^i = Y_t^I(i)$, which, using the demand function (11), can be re-written as

$$\left(\frac{P_t^i}{P_t}\right)^{-\epsilon} Y_t = Y_t^I(i)$$

Integrating over all final goods firms gives

$$\begin{aligned}
S_t^p Y_t &= Y_t^I \\
&= \left[\int_0^{(1-\overline{S})N_t} \left(Y_t^{I,i}\right)^{\frac{\epsilon^I-1}{\epsilon^I}} di\right]^{\frac{\epsilon^I}{\epsilon^I-1}} \\
&= \left[(1-\overline{S})N_t\right]^{\nu+1} Y_t^{I,i} \quad = \left[(1-\overline{S})N_t\right]^{\nu+1}\left[A_t\left(\frac{K_t}{(1-\overline{S})N_t}\right)^{\alpha}(h_t H_t)^{1-\alpha}\right] \\
&= \left[(1-\overline{S})N_t\right]^{\nu} A_t (K_t)^{\alpha}\left(h_t H_t (1-\overline{S})N_t\right)^{1-\alpha}
\end{aligned}$$

which is the standard resource constraint, adjusted for the existence of price dispersion $S_t^p \equiv \int_0^1 \left(\frac{P_t^i}{P_t}\right)^{-\epsilon} di$ and for variations in the number of intermediate good varieties (induced by the presence of unemployment/compulsory schooling).

**Aggregate profits:** An expression for the economy-wide profits is as follows,

$$\begin{aligned}
\Pi_t &= \int_0^{(1-\overline{S})N_t} \Pi_t^{I,i} di + \int_0^1 \left(\left(1-\tau_t^{\dagger}\right)\frac{P_t^i}{P_t} - mc_t\right) Y_t^i di \\
&= \left(1-\tau_t^{\dagger}-\tau_t^I p_t^I S_t^p\right) Y_t - w_t h_t H_t \left(1-\overline{S}\right) N_t - (1-\upsilon_t)\, p_t^K K_t
\end{aligned}$$

where profits in the intermediate goods sector are

$$\begin{aligned}
\int_0^{(1-\overline{S})N_t} \Pi_t^{I,i} di &= \left[\left(1-\overline{S}\right)N_t\right]\Pi_t^{I,i} \\
&= \left\{\begin{array}{c} \left(1-\tau_t^I\right)\left[\left(1-\overline{S}\right)N_t\right]^{\nu} p_t^I A_t (K_t)^{\alpha}\left(h_t H_t\left(1-\overline{S}\right)N_t\right)^{1-\alpha} - w_t h_t H_t\left(1-\overline{S}\right)N_t \\ -(1-\upsilon_t)\, p_t^K K_t \end{array}\right\} \\
&= \left(1-\tau_t^I\right) p_t^I \left(S_t^p Y_t\right) - w_t h_t H_t\left(1-\overline{S}\right)N_t - (1-\upsilon_t)\, p_t^K K_t
\end{aligned}$$

and final goods firms' profits are,

$$\begin{aligned}
\int_0^1 \left(\left(1-\tau_t^{\dagger}\right)\frac{P_t^i}{P_t} - mc_t\right) Y_t^i di &= \int_0^1 \left(\left(1-\tau_t^{\dagger}\right)\frac{P_t^i}{P_t} - mc_t\right)\left(\frac{P_t^i}{P_t}\right)^{-\epsilon} Y_t di \\
&= Y_t\left[\left(1-\tau_t^{\dagger}\right)\int_0^1\left(\frac{P_t^i}{P_t}\right)^{1-\epsilon} di - mc_t \int_0^1 \left(\frac{P_t^i}{P_t}\right)^{-\epsilon} di\right] \\
&= Y_t\left[\left(1-\tau_t^{\dagger}\right) - p_t^I S_t^p\right]
\end{aligned}$$

where we have used the fact that $mc_t = p_t^I$.

**Aggregate resource constraint:** Combining the household's budget constraint with the government budget constraint and the above definition of profits and given that in equilibrium net financial assets are zero in the absence of debt, the aggregare resource constraint is:

$$C_t + I_t + \phi\left(\frac{I_t}{K_t}\right)K_t + \varkappa_t V_t + G_t = Y_t$$

## B.2 System of non-stationary, non-linear equations

**Households**

Consumption Euler:

$$1 = \beta E_t\left(\frac{u_{c,t+1}}{u_{c,t}}\pi_{t+1}^{-1}\right)R_t$$

Physical capital accumulation:

$$K_{t+1} = \left(1-\delta^K\right)K_t + I_t$$

Investment:

$$u_{c,t}\left(1+\phi'\left(\frac{I_t}{K_t}\right)\right) = \beta E_t u_{c,t+1}\left[\begin{array}{c}\left(1-\tau_{t+1}\right)p_{t+1}^K - \phi\left(\frac{I_{t+1}}{K_{t+1}}\right) + \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}} \\ +\left(1+\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\left(1-\delta^K\right)\end{array}\right]$$

Human capital accumulation (where in equilibrium $\overline{H}_t = H_t$):

$$H_{t+1} = \left[1-\left(1-\overline{S}\right)N_t\delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right)U_t\delta^U\right]H_t + \left(1-\overline{S}\right)N_t\left(\overline{A^N}\left(\overline{e^N}\right)^{\theta^N}H_t\right) + \overline{S}\left(\overline{A^S}\left(\overline{e^S}\right)^{\theta^S}H_t\right)$$

Value of a unit of human capital:

$$\begin{aligned}\lambda_{3t} &= \beta E_t u_{c,t+1}\left(1-\tau_{t+1}\right)w_{t+1}h_{t+1}(1-\overline{S})N_{t+1} + \\ &\quad +\beta E_t\lambda_{3t+1}\left[\begin{array}{c}1-\left(1-\overline{S}\right)N_{t+1}\delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right)U_{t+1}\delta^U + \left(1-\overline{S}\right)N_{t+1}\left(\theta^N\overline{A^N}\left(\overline{e^N}\right)^{\theta^N}\right) \\ +\overline{S}\left(\theta^S\overline{A^S}\left(\overline{e^S}\right)^{\theta^S}\right)\end{array}\right]\end{aligned}$$

**Intermediate goods firms and the labour market**

The matching technology:

$$\begin{aligned}M_t &= \sigma_m(\left(1-\overline{S}\right)U_t)^{\xi}(V_t)^{1-\xi} \\ &= \sigma_m\left(\theta_t\right)^{-\xi}V_t\end{aligned}$$

Market tightness:

$$\theta_t = \frac{V_t}{\left(1-\overline{S}\right)U_t}$$

The probability of filling a vacancy:

$$z_t = \frac{M_t}{V_t} = \sigma_m\left(\theta_t\right)^{-\xi}$$

The probability of the family converting a unit of time from a state of unemployment to a state of employment:

$$s_t = \frac{M_t}{\left(1-\overline{S}\right) U_t} = \sigma_m \left(\theta_t\right)^{1-\xi}$$

The evolution of employment:

$$\left(1-\overline{S}\right) N_t = (1-\vartheta)\left(1-\overline{S}\right) N_{t-1} + M_{t-1}$$

The net value to the family of possessing a job:

$$\left(V_t^E - V_t^U\right) = (1-\tau_t) w_t h_t H_t - \varpi_t - mrs_t (h_t + \overline{e^N}) + \frac{\psi}{u_{c,t}} + E_t q_{t,t+1}\left[(1-\vartheta-s_t)\left(V_{t+1}^E - V_{t+1}^U\right)\right]$$

where $mrs_t \equiv \frac{u_{l,t}}{u_{c,t}}$ is the marginal rate of substitution between consumption and leisure and $q_{t,t+1}$ is the stochastic discount factor for real payoffs, $q_{t,t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}}$.

The value to a firm of making a match:

$$J_t = (1-\tau_t)\Pi_t^{I,i} + \Xi_t + E_t q_{t,t+1}\left[(1-\vartheta) J_{t+1}\right]$$

Job creation dynamics:

$$\frac{(1-\tau_t)\varkappa_t}{z_t} = E_t q_{t,t+1} J_{t+1}$$

The representative intermediate firm's profits:

$$\Pi_t^{I,i} = \left(1-\tau_t^I\right) x_t \left[A_t \left(\frac{K_t}{\left(1-\overline{S}\right) N_t}\right)^{\alpha} (h_t H_t)^{1-\alpha}\right] - w_t h_t H_t - (1-\upsilon_t) p_t^K \frac{K_t}{\left(1-\overline{S}\right) N_t}$$

Wage under efficient bargaining:

$$\eta J_t = (1-\eta)(V_t^E - V_t^U)$$

Hours decision under efficient bargaining:

$$mrs_t = (1-\tau_t)\left(1-\tau_t^I\right) x_t mpl_t$$

Intermediate goods firms' choice of capital:

$$\left(1-\tau_t^I\right) x_t mpk_t = \left(\frac{\epsilon^I}{\epsilon^I - 1}\right)(1-\upsilon_t) p_t^K$$

where $mpk_t = \alpha A_t \left(\frac{K_t}{(1-\overline{S})N_t}\right)^{\alpha-1} (h_t H_t)^{1-\alpha}$ is the marginal product of capital of the intermediate firm.

Intermediate goods firms' implicit pricing decision:

$$\left(1-\tau_t^I\right) x_t = \left[\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} A_t^{-1} \left(\frac{\epsilon^I}{\epsilon^I - 1}(1-\upsilon_t) p_t^K\right)^{\alpha} \left(\frac{mrs_t/H_t}{1-\tau_t}\right)^{1-\alpha}\right]$$

where

$$x_t = \left[\left(1-\overline{S}\right) N_t\right]^{\nu} p_t^I$$

**Final goods producers**

Final goods firms' pricing decision:

$$\widetilde{p}_t = \left(1-\tau_t^{\dagger}\right)^{-1}\left(\frac{\epsilon}{\epsilon-1}\right)\frac{F_t^1}{F_t^2}$$

$$where \quad : \quad F_t^1 = (1-\tau_t)\,u_{c,t}p_t^I Y_t + \omega\beta E_t\left(\pi_{t+1}^{\epsilon}F_{t+1}^1\right)$$

$$: \quad F_t^2 = (1-\tau_t)\,u_{c,t}Y_t + \omega\beta E_t\left(\pi_{t+1}^{\epsilon-1}F_{t+1}^2\right)$$

Optimal price and inflation:

$$1 = (1-\omega)\left(\widetilde{p}_t\right)^{1-\epsilon} + \omega\pi_t^{\epsilon-1}$$

Evolution of price dispersion:

$$S_t^p \equiv \int_0^1 \left(\frac{P_t^i}{P_t}\right)^{-\epsilon} di = (1-\omega)\left(\widetilde{p}_t\right)^{-\epsilon} + \omega\pi_t^{\epsilon}S_{t-1}^p$$

**Aggregate identities**

Aggregate resource constraint:

$$Y_t = C_t + I_t + \phi\left(\frac{I_t}{K_t}\right)K_t + \varkappa_t V_t + G_t$$

Aggregate output:

$$S_t^p Y_t = \left[(1-\overline{S})N_t\right]^{\nu} A_t\left(K_t\right)^{\alpha}\left(h_t H_t(1-\overline{S})N_t\right)^{1-\alpha}$$

Time constraints:

$$1 = N_t + U_t = l_t + \left(1-\overline{S}\right)N_t(h_t + \overline{e^N}) + \overline{S}\overline{e^S}$$

### B.3 Stationary Dynamic Competitive Equilibrium

We transform the relevant aggregate quantities to stationary variables by defining $x_t \equiv X_t/H_t$, where $X_t = \{K_t, Y_t,\ C_t,\ I_t, J_t, V_t^E, V_t^U, G_t\}$, and $\widetilde{x}_t \equiv X_t/H_t$, where $X_t = \left\{\Pi_t^{I,i}, \Xi_t, mrs_t, mpl_t, u_{c,t}\right\}$. In addition, $\varpi \equiv \frac{\varpi_t}{H_t}$, $\varkappa \equiv \frac{\varkappa_t}{H_t}$, $\widetilde{\lambda}_{3t} \equiv \lambda_{3t}H_t$, $\widetilde{F}_t^1 \equiv F_t^1$, $\widetilde{F}_t^2 \equiv F_t^2$, $\widetilde{mpk}_t \equiv mpk_t$, and $\gamma_t \equiv H_{t+1}/H_t$.

The time-$t$ utility and marginal utilities can be written as:

$$
\begin{aligned}
u_t &= \ln(c_t) - \varphi_0 \frac{(1-l_t)^{1+\varphi}}{1+\varphi} + \left(1-\overline{S}\right) N_t \psi + \ln(H_t) \equiv \widetilde{u}_t + \ln(H_t) \\
u_{c,t} &= C_t^{-1} = \left(c_t^{-1}\right) H_t^{-1} \equiv \widetilde{u}_{c,t} H_t^{-1} \\
u_{l,t} &= \varphi_0 (1-l_t)^{\varphi} \equiv \widetilde{u}_{l,t} \\
\\
where &: \widetilde{u}_t \equiv \ln(c_t) - \varphi_0 \frac{(1-l_t)^{1+\varphi}}{1+\varphi} + \left(1-\overline{S}\right) N_t \psi \\
&: \widetilde{u}_{c,t} \equiv c_t^{-1} \qquad and \qquad \widetilde{u}_{l,t} \equiv \varphi_0 (1-l_t)^{\varphi}
\end{aligned} \tag{39}
$$

The marginal rate of substitution between consumption and leisure can be written in terms of stationary variables as:

$$mrs_t \equiv \frac{u_{l,t}}{u_{c,t}} = \frac{\widetilde{u}_{l,t}}{\widetilde{u}_{c,t}} H_t = \left[\varphi_0 c_t (1-l_t)^{\varphi}\right] H_t = \widetilde{mrs}_t\ H_t$$

The marginal product of hours worked is

$$
\begin{aligned}
mpl_t &\equiv (1-\alpha) A_t \left(\frac{K_t}{(1-\overline{S})N_t}\right)^{\alpha} H_t^{1-\alpha} h_t^{-\alpha} = \left[(1-\alpha) A_t \left(\frac{k_t}{(1-\overline{S})N_t}\right)^{\alpha} h_t^{-\alpha}\right] H_t \\
&= \left[(1-\alpha) \frac{y_t^{I,i}}{h_t}\right] H_t = \widetilde{mpl}_t\ H_t
\end{aligned}
$$

and the marginal product of capital

$$mpk_t \equiv \alpha \frac{Y_t^{I,i}}{K_t^i} = \alpha \frac{y_t^{I,i}}{k_t^i} = \widetilde{mpk}_t$$

while the capital adjustment functions are:

$$\phi\left(\frac{i_t}{k_t}\right) = \frac{\varsigma_0}{\varsigma}\left(\frac{i_t}{k_t}\right)^{\varsigma} \qquad and \qquad \phi'\left(\frac{i_t}{k_t}\right) = \varsigma_0 \left(\frac{i_t}{k_t}\right)^{\varsigma-1}$$

The non-stationary DCE can now be rewritten in stationary form as follows:
**Households**
Consumption Euler:

$$1 = \beta E_t \left( \frac{\widetilde{u}_{c,t+1}}{\widetilde{u}_{c,t}} \pi_{t+1}^{-1} \right) \gamma_t^{-1} R_t \tag{40}$$

Capital accumulation:

$$k_{t+1}\gamma_t = (1-\delta^K)k_t + i_t \tag{41}$$

Investment:

$$\widetilde{u}_{c,t}\left(1+\phi'\left(\frac{i_t}{k_t}\right)\right)\gamma_t = \beta E_t \widetilde{u}_{c,t+1}\left[\begin{array}{c}(1-\tau_{t+1})\,p_{t+1}^K - \phi\left(\frac{i_{t+1}}{k_{t+1}}\right) + \phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)\frac{i_{t+1}}{k_{t+1}} \\ +\left(1+\phi'\left(\frac{i_{t+1}}{k_{t+1}}\right)\right)\left(1-\delta^K\right)\end{array}\right] \tag{42}$$

Human capital accumulation:

$$\gamma_t = (1 - \left(1-\overline{S}\right)N_t\delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right)U_t\delta^U) + \left(1-\overline{S}\right)N_t\overline{A^N}\left(\overline{e^N}\right)^{\theta^N} + \overline{S}\overline{A^S}\left(\overline{e^S}\right)^{\theta^S} \tag{43}$$

Value of a unit of human capital:

$$\begin{aligned}\widetilde{\lambda}_{3t}\gamma_t \quad = \quad & \beta E_t \widetilde{u}_{c,t+1}\left[(1-\tau_{t+1})\,w_{t+1}h_{t+1}\left(1-\overline{S}\right)N_{t+1}\right] \\ & +\beta E_t \widetilde{\lambda}_{3t+1}\left[\begin{array}{c}(1-\left(1-\overline{S}\right)N_{t+1}\delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right)U_{t+1}\delta^U) \\ +\left(1-\overline{S}\right)N_{t+1}\left(\theta^N\overline{A^N}\left(\overline{e^N}\right)^{\theta^N}\right) + \overline{S}\left(\theta^S\overline{A^S}\left(\overline{e^S}\right)^{\theta^S}\right)\end{array}\right]\end{aligned} \tag{44}$$

**Intermediate goods firms and the labour market**
The matching technology:

$$M_t = \sigma_m\left(\left(1-\overline{S}\right)U_t\right)^{\xi}(V_t)^{1-\xi} \tag{45}$$

Market tightness:

$$\theta_t = \frac{V_t}{\left(1-\overline{S}\right)U_t} \tag{46}$$

The probability of filling a vacancy:

$$z_t = \frac{M_t}{V_t} = \sigma_m(\theta_t)^{-\xi} \tag{47}$$

The probability of the family converting a unit of time from a state of unemployment to a state of employment:

$$s_t = \frac{M_t}{\left(1-\overline{S}\right)U_t} = \sigma_m(\theta_t)^{1-\xi} \tag{48}$$

The evolution of employment:

$$\left(1-\overline{S}\right)N_t = (1-\vartheta)\left(1-\overline{S}\right)N_{t-1} + M_{t-1} \tag{49}$$

The net value to the family of possessing a job:

$$\left(v_t^E - v_t^U\right) = (1-\tau_t)w_t h_t - \varpi - \widetilde{mrs}_t \left(h_t + \overline{e^N}\right) + \frac{\psi}{\widetilde{u}_{c,t}} + \beta E_t \frac{\widetilde{u}_{c,t+1}}{\widetilde{u}_{c,t}} \left[(1-\vartheta - s_t)\left(v_{t+1}^E - v_{t+1}^U\right)\right] \tag{50}$$

The value to a firm of making a match:

$$j_t = (1-\tau_t)\,\widetilde{\Pi}_t^{I,i} + \widetilde{\Xi}_t + \beta E_t \frac{\widetilde{u}_{c,t+1}}{\widetilde{u}_{c,t}} \left[(1-\vartheta) j_{t+1}\right] \tag{51}$$

Job creation dynamics:

$$\frac{(1-\tau_t)\,\varkappa}{z_t} = \beta E_t \frac{\widetilde{u}_{c,t+1}}{\widetilde{u}_{c,t}} j_{t+1}$$

The representative intermediate firm's profits:

$$\widetilde{\Pi}_t^{I,i} = \left(1-\tau_t^I\right) x_t \left[A_t \left(\frac{k_t}{\left(1-\overline{S}\right) N_t}\right)^{\alpha} h_t^{1-\alpha}\right] - w_t h_t - (1-\upsilon_t)\, p_t^K \frac{k_t}{\left(1-\overline{S}\right) N_t} \tag{52}$$

Wage under efficient bargaining:

$$\eta j_t = (1-\eta)\left(v_t^E - v_t^U\right) \tag{53}$$

Hours decision under efficient bargaining:

$$\widetilde{mrs}_t = (1-\tau_t)\left(1-\tau_t^I\right) x_t \widetilde{mpl}_t \tag{54}$$

Intermediate goods firms' choice of capital:

$$\left(1-\tau_t^I\right) x_t \widetilde{mpk}_t = \left(\frac{\epsilon^I}{\epsilon^I - 1}\right)(1-\upsilon_t)\, p_t^K \tag{55}$$

Intermediate goods firms' implicit pricing decision:

$$\left(1-\tau_t^I\right) p_t^I = \left[(1-\overline{S})N_t\right]^{-\nu} \left[\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} A_t^{-1} \left(\frac{\epsilon^I}{\epsilon^I - 1}(1-\upsilon_t)\, p_t^K\right)^{\alpha} \left(\frac{\widetilde{mrs}_t}{1-\tau_t}\right)^{1-\alpha}\right] \tag{56}$$

where

$$x_t = \left[\left(1-\overline{S}\right) N_t\right]^{\nu} p_t^I$$

**Final goods producers**

Final goods firms' pricing decision:

$$\widetilde{p}_t = \left(1-\tau_t^{\dagger}\right)^{-1} \left(\frac{\epsilon}{\epsilon - 1}\right) \frac{\widetilde{F}_t^1}{\widetilde{F}_t^2} \tag{57}$$

$$where : \widetilde{F}_t^1 = (1-\tau_t)\,\widetilde{u}_{c,t} p_t^I y_t + \omega\beta E_t \left(\pi_{t+1}^{\epsilon} \widetilde{F}_{t+1}^1\right) \tag{58}$$

$$: \widetilde{F}_t^2 = (1-\tau_t)\,\widetilde{u}_{c,t} y_t + \omega\beta E_t \left(\pi_{t+1}^{\epsilon-1} \widetilde{F}_{t+1}^2\right) \tag{59}$$

Optimal price and inflation:

$$1 = (1 - \omega) \left(\widetilde{p}_t\right)^{1-\epsilon} + \omega \pi_t^{\epsilon-1} \tag{60}$$

Evolution of price dispersion:

$$S_t^p \equiv \int_0^1 \left(\frac{P_t^i}{P_t}\right)^{-\epsilon} di = (1 - \omega) \left(\widetilde{p}_t\right)^{-\epsilon} + \omega \pi_t^{\epsilon} S_{t-1}^p \tag{61}$$

**Aggregate identities and exogenous processes**

National accounting identity:

$$c_t + i_t + \phi \left(\frac{i_t}{k_t}\right) k_t + \varkappa V_t + g_t = y_t \tag{62}$$

Aggregate resource constraint:

$$S_t^p y_t = \left[(1 - \overline{S}) N_t\right]^{\nu} A_t k_t^{\alpha} \left(h_t \left(1 - \overline{S}\right) N_t\right)^{1-\alpha} \tag{63}$$

Time constraints:

$$1 = N_t + U_t = l_t + \left(1 - \overline{S}\right) N_t (h_t + \overline{e^N}) + \overline{S}\overline{e^S} \tag{64}$$

Stationary exogenous government spending process:

$$\ln g_t = (1 - \rho_G) \ln \overline{g} + \rho_G \ln g_{t-1} + \varepsilon_t^G \tag{65}$$

Stationary exogenous technology process:

$$\ln A_t = (1 - \rho_A) \ln A + \rho_A \ln A_{t-1} + \varepsilon_t^A \tag{66}$$

## C The Social Planner's Problem

We consider a version of the social planner's problem, where the planner is constrained by the workings of the labour market (see, for example, Tomas (2008), Arseneau and Chugh (2009), or Faia (2009)). This delivers a constrained efficient allocation, which allows for a measure of the 'best' employment level. The social planner chooses $\left\{C_t^*, K_{t+1}^*, I_t^*, H_{t+1}^*, h_t^*, N_t^*, V_t^*\right\}$ so as to maximise the utility of the representative household,

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^*, l_t^*, N_t^*\right)$$

subject to the capital evolution equation

$$K_{t+1}^* = (1-\delta^K)K_t^* + I_t^*$$

the human capital evolution equation

$$H_{t+1}^* = \left[1-(1-\overline{S})N_t^*\delta^N - \overline{S}\delta^N - (1-\overline{S})U_t^*\delta^U\right] H_t^* + \left(1-\overline{S}\right) N_t^* \left(\overline{A^N}\left(\overline{e^N}\right)^{\theta^N} H_t^*\right) + \overline{S}\left(\overline{A^S}\left(\overline{e^S}\right)^{\theta^S} H_t^*\right)$$

the evolution of employment

$$(1-\overline{S})N_t^* = (1-\vartheta)(1-\overline{S})N_{t-1}^* + M_{t-1}^*$$

and the aggregate resource constraint

$$C_t^* + I_t^* + \phi\left(\frac{I_t^*}{K_t^*}\right) K_t^* + \left(\varkappa H_t^*\right) V_t^* + \overline{g} H_t^* = Y_t^*$$

all accounting for the labour market specification, where households must pay vacancy posting costs. The social planner is also constrained by the assumptions that effort devoted to the accumulation of human capital is constant $\left(e_t^N = \overline{e^N}\right)$ and that vacancy-posting costs and government expenditures grow over time in line with the level of human capital ($\varkappa_t^* = \varkappa H_t^*,\ G_t^* = \overline{g} H_t^*$).

Total output is the CES aggregate of the intermediate goods produced by the $(1-\overline{S})N_t^*$ identical firms ($Y_t^{i*}$ denoting the output of each producing enterprise)

$$\begin{aligned} Y_t^* &= \left[(1-\overline{S})N_t^*\right]^{\nu - \frac{1}{\epsilon^I - 1}} \left(\int_0^{(1-\overline{S})N_t^*} \left(Y_t^{i*}\right)^{\frac{\epsilon^I - 1}{\epsilon^I}} di\right)^{\frac{\epsilon^I}{\epsilon^I - 1}} = \left[(1-\overline{S})N_t^*\right]^{\nu+1} Y_t^{i*} \\ &= \left[(1-\overline{S})N_t^*\right]^{\nu} \left[A_t \left(K_t^*\right)^{\alpha} \left(h_t^* H_t^* (1-\overline{S})N_t^*\right)^{1-\alpha}\right] \end{aligned}$$

The matching technology is $M_t^* = \sigma_m((1-\overline{S})U_t^*)^{\xi}(V_t^*)^{1-\xi} = \sigma_m\left(\theta_t^*\right)^{-\xi} V_t^*$, where $\theta_t^* = \frac{V_t^*}{(1-\overline{S})U_t^*}$ is the market tightness measure, unemployment is $U_t^* = 1 - N_t^*$, and leisure is $l_t^* = 1 - \left(1-\overline{S}\right) N_t^* \left(h_t^* + \overline{e^N}\right) - \overline{S}\left(\overline{e^S}\right)$.

The Lagrangian is

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \beta^t \{ u\left(C_t^*, l_t^*, N_t^*\right) - \\
&\quad -\lambda_{1t}^* \left[ C_t^* + I_t^* + \phi\left(\frac{I_t^*}{K_t^*}\right) K_t^* + \left(\varkappa H_t^*\right) V_t^* + \overline{g} H_t^* - Y_t^* \right] \\
&\quad -\lambda_{2t}^* [K_{t+1}^* - (1-\delta^K) K_t^* - I_t^*] \\
&\quad -\lambda_{3t}^* \left[ \begin{array}{c} H_{t+1}^* - (1-(1-\overline{S}) N_t^* \delta^N - \overline{S}\delta^N - (1-\overline{S}) U_t^* \delta^U) H_t^* - (1-\overline{S}) N_t^* \left( \overline{A^N} \left(\overline{e^N}\right)^{\theta^N} H_t^* \right) \\ -\overline{S} \left( \overline{A^S} \left(\overline{e^S}\right)^{\theta^S} H_t^* \right) \end{array} \right] \\
&\quad -\lambda_{4t}^* \left[ (1-\overline{S}) N_t^* - (1-\vartheta)(1-\overline{S}) N_{t-1}^* - M_{t-1}^* \right] \}
\end{aligned}
$$

The first order conditions are:
Hours worked $(h_t^*)$ :

$$
\begin{aligned}
\frac{u_{l,t}^*}{u_{c,t}^*} &= \left[ (1-\alpha) \frac{Y_t^*}{h_t^*} \right] \frac{1}{(1-\overline{S}) N_t^*} \\
&\implies \quad mrs_t^* = \left[ (1-\overline{S}) N_t^* \right]^{\nu} mpl_t^*
\end{aligned}
$$

where $mrs_t^* \equiv \frac{u_{l,t}^*}{u_{c,t}^*}$ is the marginal rate of substitution between consumption and leisure and $mpl_t^* \equiv (1-\alpha) \frac{Y_t^{i*}}{h_t^*}$ is the marginal product of labour of the individual firm.

Physical capital $\left(K_{t+1}^*\right)$ :

$$
u_{c,t}^* \left( 1 + \phi'\left(\frac{I_t^*}{K_t^*}\right) \right) = \beta E_t u_{c,t+1}^* \left[ \begin{array}{c} \alpha \frac{Y_{t+1}^*}{K_{t+1}^*} - \phi\left(\frac{I_{t+1}^*}{K_{t+1}^*}\right) + \phi'\left(\frac{I_{t+1}^*}{K_{t+1}^*}\right) \frac{I_{t+1}^*}{K_{t+1}^*} \\ + \left( 1 + \phi'\left(\frac{I_{t+1}^*}{K_{t+1}^*}\right) \right) (1-\delta^K) \end{array} \right]
$$

where we can write $\alpha \frac{Y_t^*}{K_t^*} = \left[ (1-\overline{S}) N_t^* \right]^{\nu} mpk_t^*$, with $mpk_t^* \equiv \alpha \frac{Y_t^{i*}}{K_t^{i*}}$ defined as the marginal product of capital.

Human capital $\left(H_{t+1}^*\right)$ :

$$
\lambda_{3t}^* = \beta E_t u_{c,t+1}^* \left[ (1-\alpha) \frac{Y_{t+1}^*}{H_{t+1}^*} - \varkappa V_{t+1}^* - \overline{g} \right] + \beta E_t \lambda_{3t+1}^* \left[ \begin{array}{c} 1 - \left(1-\overline{S}\right) N_{t+1}^* \delta^N - \overline{S}\delta^N - \left(1-\overline{S}\right) U_{t+1}^* \delta^U \\ + (1-\overline{S}) N_{t+1}^* \overline{A^N} \left(\overline{e^N}\right)^{\theta^N} + \overline{S}\,\overline{A^S} \left(\overline{e^S}\right)^{\theta^S} \end{array} \right]
$$

Vacancies $(V_t^*)$ :

$$
u_{c,t}^* \varkappa_t = \beta E_t \lambda_{4t+1}^* \left[ (1-\xi)\, \sigma_m \left(\theta_t^*\right)^{-\xi} \right]
$$

Employment $(N_t^*)$ :

$$\lambda_{4t}^* = u_{c,t}^* \left[ (\nu+1-\alpha) \frac{Y_t^*}{\left(1-\overline{S}\right) N_t^*} \right] - u_{l,t}^* \left( h_t^* + \overline{e^N} \right) + \psi - \lambda_{3t}^* \left[ \delta^N - \delta^U - \overline{A^N} \left( \overline{e^N} \right)^{\theta^N} \right] H_t^*$$
$$+ \beta E_t \lambda_{4t+1}^* \left[ 1 - \vartheta - \xi \sigma_m \left(\theta_t^*\right)^{1-\xi} \right]$$

Combining the first order conditions for vacancies and employment, equations (**??**) and (**??**), we obtain the efficient job creation condition:

$$\frac{\varkappa}{\sigma_m \left(\theta_t^*\right)^{-\xi}} = \beta E_t \frac{u_{c,t+1}^*}{u_{c,t}^*} \left\{ \begin{array}{c} (1-\xi) \left[ \begin{array}{c} (\nu+1-\alpha) \frac{Y_{t+1}^*}{(1-\overline{S}) N_{t+1}^*} - mrs_{t+1}^* \left( h_{t+1}^* + \overline{e^N} \right) + \frac{\psi}{u_{c,t+1}^*} \\ - \frac{\lambda_{3t+1}^* H_{t+1}^*}{u_{c,t+1}^*} \left( \delta^N - \delta^U - \overline{A^N} \left( \overline{e^N} \right)^{\theta^N} \right) \end{array} \right] \\ + \left[ 1 - \vartheta - \xi \sigma_m \left(\theta_{t+1}^*\right)^{1-\xi} \right] \frac{\varkappa_{t+1}}{\sigma_m \left(\theta_{t+1}^*\right)^{-\xi}} \end{array} \right\}$$

### C.1 The stationary representation

To transform the relevant aggregate quantities to stationary variables, we define $x_t^* \equiv X_t^*/H_t^*$, where $X_t^* = \{K_t^*, C_t^*, I_t^*, Y_t^*\}$, and $\widetilde{x}_t \equiv X_t/H_t$, where $X_t = \{mrs_t, mpl_t, mpk_t, u_{c,t}\}$. In addition, $\widetilde{\lambda}_{3t}^* \equiv \lambda_{3t}^* H_t^*$, $\widetilde{\lambda}_{4t}^* \equiv \lambda_{4t}^*$ and $\gamma_t^* \equiv H_{t+1}^*/H_t^*$. The non-stationary solution to the social planner's problem can now be rewritten in stationary form as follows:

Physical capital accumulation

$$k_{t+1}^* \gamma_t^* = (1-\delta^K) k_t^* + i_t^* \tag{67}$$

Human capital evolution equation

$$\gamma_t^* = (1-(1-\overline{S}) N_t^* \delta^N - \overline{S} \delta^N - (1-\overline{S}) U_t^* \delta^U) + \overline{A^N} (\overline{e^N})^{\theta^N} (1-\overline{S}) N_t^* + \overline{S}\,\overline{A^S} \left( \overline{e^S} \right)^{\theta^S} \tag{68}$$

Evolution of employment

$$(1-\overline{S}) N_t^* = (1-\vartheta)(1-\overline{S}) N_{t-1}^* + M_{t-1}^* \tag{69}$$

Aggregate resource constraint,

$$c_t^* + i_t^* + \phi \left( \frac{i_t^*}{k_t^*} \right) k_t^* + \varkappa V_t^* + \overline{g} = y_t^* \tag{70}$$

where

$$y_t^* = \left[ (1-\overline{S}) N_t^* \right]^{\nu} \left[ A_t \left(k_t^*\right)^{\alpha} \left( h_t^* (1-\overline{S}) N_t^* \right)^{1-\alpha} \right] \tag{71}$$

Hours worked:

$$\widetilde{mrs}_t^* = \left[ (1-\overline{S}) N_t^* \right]^{\nu} \widetilde{mpl}_t^* \tag{72}$$

Investment:

$$\widetilde{u}_{c,t}^{*}\left(1+\phi^{'}\left(\frac{i_t^*}{k_t^*}\right)\right)\gamma_t^*=\beta E_t\widetilde{u}_{c,t+1}^{*}\left[\begin{array}{c}\left[(1-\overline{S})N_t^*\right]^{\nu}\widetilde{mpk}_t^{*}-\phi\left(\frac{i_{t+1}^*}{k_{t+1}^*}\right)+\phi'\left(\frac{i_{t+1}^*}{k_{t+1}^*}\right)\frac{i_{t+1}^*}{k_{t+1}^*}\\ +\left(1+\phi'\left(\frac{i_{t+1}^*}{k_{t+1}^*}\right)\right)\left(1-\delta^K\right)\end{array}\right] \tag{73}$$

Human capital:

$$\widetilde{\lambda}_{3t}^{*}\gamma_t^*=\beta E_t\widetilde{u}_{c,t+1}^{*}\left[(1-\alpha)\,y_{t+1}^*-\varkappa V_{t+1}^*-\overline{g}\right]+\beta E_t\widetilde{\lambda}_{3t+1}^{*}\gamma_{t+1}^* \tag{74}$$

Job creation condition:

$$\frac{\varkappa}{\sigma_m\left(\theta_t^*\right)^{-\xi}}=\beta E_t\frac{\widetilde{u}_{c,t+1}^{*}}{\widetilde{u}_{c,t}^{*}}\left\{\begin{array}{c}(1-\xi)\left[\begin{array}{c}(\nu+1-\alpha)\frac{y_{t+1}^*}{(1-\overline{S})N_{t+1}^*}-\widetilde{mrs}_{t+1}^{*}\left(h_{t+1}^*+\overline{e^N}\right)+\frac{\psi}{\widetilde{u}_{c,t+1}^{*}}\\ -\frac{\widetilde{\lambda}_{3t+1}^{*}}{\widetilde{u}_{c,t+1}^{*}}\left(\delta^N-\delta^U-\overline{A^N}\left(\overline{e^N}\right)^{\theta^N}\right)\end{array}\right]\\ +\left[1-\vartheta-\xi\sigma_m\left(\theta_{t+1}^*\right)^{1-\xi}\right]\frac{\varkappa}{\sigma_m\left(\theta_{t+1}^*\right)^{-\xi}}\end{array}\right\} \tag{75}$$