Bubbles and Trading Volume

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Abstract

Rational bubbles in stocks can cause increases in trading volume, even after accounting for their expansionary effect on output and other macroeconomic aggregates. Trading volume increases are not caused by speculation driven by differences in beliefs. Dividend-bearing assets used to transfer resources intertemporally reduce the need for portfolio adjustment after a bad shock. Bubbles, on the contrary, do not produce dividends and require more rebalancing after a bad shock.

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1 Introduction

Episodes of large stock market run-ups followed by abrupt crashes, without matching movements in fundamentals, are referred to as bubbles. As the recent experience of US and Japan attests, these movements in asset prices are closely tracked by macroeconomic aggregates, such as household wealth, output, consumption and investment (Martin & Ventura 2012). They are also accompanied by large increases and subsequent collapse in trading volume (Cochrane 2002).

Formally, a (rational) bubble is defined as the price of an asset in excess of its fundamental value, computed as the discounted (at market rates) present value of dividends. Recent overlapping generations models of rational bubbles, based on Tirole (1985), explained some of the connections between bubbles and macroeconomic aggregates (Martin & Ventura 2012, Farhi & Tirole 2012). However, such models are not appropriate (or attempt) to analyze the trading volume effects of bubbles, due to infrequent, intergenerational trading only.

This paper shows that rational bubbles can indeed expand the trading volume, in addition to increasing output, consumption, labor supply and welfare, in a Bewley style, deterministic economy, with elastic labor supply. The model is an extension of the one analyzed in Kocherlakota (2011), or alternatively, is a tractable particular case of the model studied computationally in Guerrieri & Lorenzoni (2011), but with long-lived assets and bubbles.

First, bubbles are expansionary and expand trading because they create wealth effects and inject liquidity, by relaxing the borrowing constraints. The general equivalence of bubbles to relaxations of agents' debt limits was shown by Kocherlakota (2008) for Arrow-complete markets and redundant long-lived assets, and by Bejan & Bidian (2014), for incomplete or only dynamically complete markets.

Second, and more surprisingly, even when comparing a bubbly economy to the equivalent bubble-free economy with relaxed debt limits, bubbles nevertheless produce increases in trading volumes, as agents adjust their portfolios in response to their presence. For this effect to arise, it is crucial that the bubble is attached to a dividend-paying asset. As bubbles do not pay dividends, they require more rebalancing in response to a bad shock.

To my knowledge, this is the first paper linking rational bubbles to trading volume increases. There is a large literature on *speculative bubbles* in economies with short sale constraints and heterogeneous beliefs (Harrison & Kreps 1978, Morris 1996), which arose because (Scheinkman & Xiong 2003)

> "rational bubble models are incapable of connecting bubbles with turnover."

These papers use partial equilibrium models, in which infinitely wealthy risk neutral agents exchange the asset, the pessimists selling it to the optimists. Beliefs are constructed such that agents take turns in being the optimists, which results in frequent speculative trading. This results in a speculative component in prices, or a "speculative bubble", under a relaxed definition of fundamental value, taken there to be the maximum amount that an agent would be willing to pay when forced to maintain the holdings of the asset forever. However, the price of the asset is in fact equal with the present value of its dividends discounted at market rates.

In addition to the special definition, speculative bubbles suffers from various limitations. First, they do not cause wealth or liquidity effects, and are not related to macro aggregates such as consumption, output, interest rates. Second, if learning is allowed, agents' beliefs converge and the speculative component (bubble) disappears (Morris 1996). Third, as explained in Scheinkman & Xiong (2003), it is also hard to generate realistic time series dynamics for speculative bubbles.

The model here does not require heterogeneous beliefs and shows that rational bubbles can generate increases in trading volume, despite the claims of the aforementioned literature, in addition to affecting the macroeconomic aggregates. With heterogeneous beliefs, there would be trading for speculative, in addition to hedging (risk-sharing) reasons, leading to larger trading volumes. Adding the speculative component in prices to a rational bubble would lead also to larger measured "bubbles".

2 Model

There are two agents $\{1,2\}$ with identical utilities $\sum_{t\geq 0}\beta^t\left(\ln c_t - n_t\right)$ over consumption (c) and labor (n), where $0 < \beta < 1$. Agent 1 is productive in odd periods $\{1,3,\ldots\}$, while agent 2 is productive in even periods $\{0,2,4,\ldots\}$. There is a production technology that allows the conversion of labor into output. Agent $i \in \{1,2\}$ can produce z_t^i units of consumption per unit of labor, where $z_t^i = 1$ if i is productive at t, and 0 otherwise. Additionally, at each date t, agent i has an endowment of goods e_t^i, where $e_t^i = y^H$(respectively, $e_t^i = y^L$) if i is productive (respectively, unproductive) at t, with $y^H \leq 1$, $y^L < \beta$.

2.1 Bond equilibrium

Agents can trade in bonds in zero supply, and initially have no endowment of them. For a date t, let b_t^i be the bonds acquired by i at a price q_t. Agents face debt limits B, with $0 < B \leq \frac{1-y^L}{1+\beta}$. Budget and debt constraints are

$$c_t^i + q_t b_t^i = e_t^i + z_t^i n_t^i + b_{t-1}^i,\ b_{t-1}^i \geq -B,\ \forall t \geq 0. \tag{2.1}$$

Let $R_{t+1} := 1/q_t$ be the gross interest rate from date t to $t+1$.

In equilibrium, unproductive agents are borrowing-constrained. The transition to the steady state is complete after the initial period. At date 0 (during transition), the productive (unproductive) agent consumes c_0^H and works n_0 (0), and the interest rate is R_1. At any date $t \geq 1$, the productive (unproductive) agent has consumption c_t^i equal to c^H (c^L), bond holdings b_{t-1}^i equal to $-B$ (B), his labor supply n_t^i is n (0) and the interest rates are constant, R. Equilibrium variables are determined from agents' first order conditions

(FOCs) and budget constraints:

$$\frac{1}{c^H} = 1; \quad \frac{c^L}{c^H} = \beta R; \quad \frac{c^H}{c^L} \geq \beta R, \tag{2.2}$$

$$\frac{1}{c_0^H} = 1; \quad \frac{c^L}{c_0^H} = \beta R_1; \quad \frac{c^H}{c_0^L} \geq \beta R_1, \tag{2.3}$$

$$c^H + R^{-1}B = y^H - B + n, \quad c^L - R^{-1}B = y^L + B, \tag{2.4}$$

$$c_0^H + R_1^{-1}B = y^H + n_0, \quad c_0^L - R_1^{-1}B = y^L. \tag{2.5}$$

Thus, R is an increasing function of B ($R = R(B)$ increasing), as it is the unique solution in $\left(\frac{y^L}{\beta}, \frac{1}{\beta}\right]$ of $\beta R = y^L + B(1 + R^{-1})$ $(= c^L)$, or equivalently of

$$B = \frac{\beta R - y^L}{1 + R^{-1}}. \tag{2.6}$$

Indeed, the right hand side of (2.6) is strictly increasing in R, and equal to 0 for $R = y^L/\beta$ and to $\frac{1-y^L}{1+\beta}$ for $R = \frac{1}{\beta}$. Consumption, labor supply and interest rates during transition are obtained as function of R from (2.2)-(2.5),

$$c^H = c_0^H = 1, \; R_1 = R, \; c^L = \beta R, \; c_0^L = y^L + R^{-1}B, \tag{2.7}$$

$$n = 1 + B + R^{-1}B - y^H, \; n_0 = 1 + R^{-1}B - y^H. \tag{2.8}$$

It remains to verify that the FOCs of unproductive agents hold, that transversality and market clearing conditions hold, and that consumption and labor supply are positive. The inequalities in (2.2)-(2.3) hold if $c^H \geq c^L$, or equivalently, if $R \leq 1/\beta$, which is true, by (2.1) (and (2.6)). Consumption and labor supply are positive by (2.7) and (2.8), since $y^H \leq 1$ by assumption. The transversality conditions[1] and the market clearing conditions are satisfied:

$$\lim_{t\to\infty} \frac{\beta^t}{c_t^i}(b_{t-1}^i + B) = 0, \ \theta_t^1 + \theta_t^2 = 0,$$
$$c^H + c^L = y^H + y^L + n, \ c_0^H + c_0^L = y^H + y^L + n_0.$$

2.2 Consequences of a credit crunch

Notice that consumption in steady state is increasing in R, and hence in B (by (2.6)), as $c^L = \beta R$ and $c^H = 1$. Similarly, consumption during transition, labor supply (both during transition and in steady state) and interest rates are all increasing functions of B (and of R, by (2.6)). Indeed, they are increasing functions of $R^{-1}B$, and therefore of B , since

$$R^{-1}B = \beta - \frac{\beta + y^L}{1 + R}. \tag{2.9}$$

Intuitively, a credit crunch (a decrease in B) lowers the interest rates, as all agents want to save more. Constrained borrowers have to reduce their indebtedness, while unconstrained ones increase their savings for precautionary reasons. The borrowing constrained unproductive agents adjust by consuming less as they cannot work more, while the productive agents reduce their labor supply due to the low return on saving. As shown computationally

[1] See Bidian & Bejan (2015) for their derivation.

(for plausible parametrizations) in Guerrieri & Lorenzoni (2011), even when indebted agents can adjust by both spending less and working more, the consumption side dominates and output falls.

The welfare of the agents is also lower after a credit crunch, since the utilities of the first agent (initially unproductive) and second agent (initially productive) are

$$
\begin{aligned}
U^L &:= \ln c_0^L + \frac{\beta}{1-\beta^2}\left(\ln c^H - n + \beta \ln c^L\right) \\
&= \ln(y^L + R^{-1}B) + \frac{\beta^2}{1-\beta^2}(\ln R - R) + \frac{\beta}{1-\beta^2}(\beta \ln \beta + y^L + y^H - 1), \\
U^H &:= \ln c_0^H + \frac{\beta}{1-\beta^2}\left(\ln c^L - \beta n\right) = \frac{\beta}{1-\beta^2}\left(\ln R - \beta^2 R + \ln \beta + \beta y^L\right),
\end{aligned}
$$

and therefore are increasing functions of R (and B).[2]

Finally, trading volume decreases in the aftermath of a credit crunch. Indeed, by (3.1), the trading volume at t is $|b_{t-1}^i - q_t b_t^i| = B + R^{-1}B$, and therefore decreases when B decreases.

For the rest of the paper, I focus on the equilibrium with zero net interest

[2]Clearly $\frac{\partial U^H}{\partial R} > 0$ as $R \le 1/\beta$. Similarly,

$$
\frac{\partial U^L}{\partial R} = \frac{\beta + y^L}{y^L + R^{-1}B}\frac{1}{(1+R)^2} + \frac{\beta^2}{1-\beta^2}\left(\frac{1}{R} - 1\right),
$$

which is a decreasing function of R. Moreover, $\left.\frac{\partial U^L}{\partial R}\right|_{R=1/\beta} = 0$, therefore $\frac{\partial U^L}{\partial R} > 0$ for $R < 1/\beta$.

rates, $R = \bar{R} := 1$, which by (2.6), is associated to the debt limits[3]

$$\bar{B} := \frac{\beta - y^L}{2}. \tag{2.10}$$

3 Bubbles

Bubbles require the presence of long-lived assets. Assume that, instead of bonds, agents can trade in an asset which, at each date t, has dividends $d_t := \lambda \eta^t$ and an ex-dividend price p_t, where $0 \leq \eta < 1$ and $\lambda > 0$. If $\eta = 0$, the asset does not pay dividends, and will be referred to as fiat money. If $\eta > 0$, the asset can be interpreted as a unit of capital, depreciating at rate $1 - \eta$, and paying dividends λ per unit, each period. Denote by θ_t^i the asset holdings of i at $t \geq 0$. Let θ_{-1}^i be the initial endowment of the asset for i. The budget constraints of agent i are

$$c_t^i + p_t \theta_t^i = e_t^i + z_t^i n_t^i + (p_t + d_t)\theta_{t-1}^i, \quad \forall t \geq 0.$$

Let $R_{t+1} = (p_{t+1} + d_{t+1})/p_t$ be the return on the asset. By iteration,

$$p_t = \sum_{\tau > t} \prod_{s=t+1}^{\tau} R_s^{-1} d_\tau + \lim_{\tau \to \infty} \prod_{s=t+1}^{\tau} R_s^{-1} p_\tau.$$

[3] As known from Hellwig & Lorenzoni (2009) and Bidian & Bejan (2015), these are the endogenous debt limits that prevent default and allow for maximal credit expansion, when the penalty for default is an interdiction to borrow. Other penalties for default are studied in Bidian (2015).

The term $\lim_{\tau\to\infty}\prod_{s=t+1}^{\tau} R_s^{-1} p_\tau$ represents a *bubble* at t whenever it is non-zero. It is the part of the asset's price in excess of the present value of its future dividends.

3.1 Wealth effects of bubbles

In this section I focus on the fiat money case ($\eta = 0$). The initial endowments of the asset are $\theta_{-1}^1 = \theta_{-1}^2 = \frac{1}{2}$. A positive price $p_t > 0$ for money represents a bubble. Let $0 < \varepsilon < 2\bar{B}$, with $\bar{B}$ given by (2.10), and assume that the economy experienced a credit crunch and agents face debt limits $\bar{B} - \varepsilon/2$:

$$p_t \theta_{t-1}^i \geq -(\bar{B} - \varepsilon/2).$$

There exists a bubbly equilibrium with money valued at ε ($p_t = \varepsilon$ for all t) equivalent (from the point of view of allocations and interest rates) to the bond equilibrium of Section 2.1, where agents were subject to the more relaxed debt limits $\bar{B}$. The bubble of size ε boosts the initial wealth of i by $\theta_{-1}^i \varepsilon = \varepsilon/2$. The tighter future debt limits (reduced by $\varepsilon/2$) force the agents to save the additional wealth, sterilizing the windfall created by the bubble. This equivalence is an instance of the "bubble equivalence theorem" of Kocherlakota (2008) (for Arrow-complete markets and redundant long-lived assets) and of Bejan & Bidian (2014) (for incomplete markets or for dynamically complete markets, as is the case here). The bubble is expansionary, as it counteracts the effect of the credit crunch (the tightening of agents' debt limits by $\varepsilon/2$), through the wealth injection to the asset holders.

Portfolio (money) holdings θ_t^H (for the productive agent at t) and θ_t^L (for the unproductive) are

$$\theta_t^L = -(\bar{B} - \varepsilon/2)/p_{t+1} = 1/2 - \bar{B}/\varepsilon, \quad \theta_t^H = 1/2 + \bar{B}/\varepsilon. \tag{3.1}$$

The trading volume in the bubbly equilibrium with debt limits $\bar{B} - \varepsilon/2$ is equal to the one in the equivalent bond equilibrium with debt limits $\bar{B}$ (where volume was $\bar{B} + \bar{R}^{-1}\bar{B} = 2\bar{B}$):

$$|p_t(\theta_t^i - \theta_{t-1}^i)| = 2\bar{B}.$$

It follows that a bubble in a non-dividend paying asset, while increasing trading due to its wealth effect and the injection of liquidity (relaxation of debt limits), it cannot increase the trading volume beyond the level that occurs in the equivalent bubble-free equilibrium with appropriately relaxed debt limits.

3.2 Portfolio rebalancing effects of bubbles

Surprisingly, a bubble in a dividend-paying asset leads to a higher trading volume even when compared to the equivalent (in terms of allocations) bubble-free equilibrium with more relaxed debt limits, that is, after the wealth effect of the bubble is accounted for. Dividend-bearing assets used to transfer resources intertemporally reduce the need for portfolio adjustment after a bad shock. Bubbles, by not providing dividends, require more rebalancing after a bad shock.

The most transparent to illustrate this mechanism is to remove the wealth effects of bubbles, by analyzing first a zero supply asset. This assumption will be dropped later. Consider a dividend-paying asset ($\eta > 0$), in zero supply, $\theta^1_{-1} = \theta^2_{-1} = 0$. Agents are subject to debt limits $\bar{B}$ (see (2.10)).

The bond equilibrium allocations under debt limits $\bar{B}$ can be achieved by trading in the asset instead. In the absence of a bubble, asset prices p_t and portfolios θ^H_t, θ^L_t are

$$p_t = \sum_{s>t} \bar{R}^{-(s-t)} d_s = \frac{\lambda \eta^{t+1}}{1-\eta}, \quad \theta^H_t = -\theta^L_t = \frac{\bar{B}}{p_{t+1} + d_{t+1}}, \ \forall t \geq 0. \tag{3.2}$$

Using (3.2), the trading volume at t is

$$|p_t(\theta^i_t - \theta^i_{t-1})| = \begin{cases} \bar{R}^{-1}\bar{B} \quad (= \bar{B}) & \text{if} \quad t = 0 \\ \bar{R}^{-1}\bar{B}(1+\eta) \quad (= \bar{B}(1+\eta)) & \text{if} \quad t > 0 \end{cases}. \tag{3.3}$$

A bubble in such a zero-supply asset produces no wealth effects and leaves allocations unchanged. The bubble-free equilibrium is equivalent with a hatted (bubbly) equilibrium, where the asset prices are higher by $\varepsilon > 0$ ($\hat{p}_t = p_t + \varepsilon$), the allocations and debt limits are unchanged, and agents adjust their portfolios in the presence of altered prices to maintain the same wealth transfers:

$$\hat{\theta}_t^i = \frac{\theta_t^i}{1+\Lambda_t}, \text{ where } \Lambda_t = \frac{\varepsilon}{p_t}, \quad \forall t \geq 0, i \in \{1,2\}. \tag{3.4}$$

This is an application of the bubble equivalence theorem of Bejan & Bidian (2014), or it can be verified directly. In contrast to the fiat money equilibrium, the bubble has no wealth effects, since the asset is in zero supply. Using $\theta_{t-1}^i = -\eta\theta_t^i$ and $\Lambda_{t-1} = \eta\Lambda_t$, in the bubbly equilibrium the trading volume is

$$\begin{aligned} |\hat{p}_t(\hat{\theta}_t^i - \hat{\theta}_{t-1}^i)| = p_t(1+\Lambda_t)\left|\frac{\theta_t^i}{1+\Lambda_t} - \frac{\theta_{t-1}^i}{1+\Lambda_{t-1}}\right| &= |p_t\theta_t^i| \cdot \left(1 + \frac{\eta(1+\Lambda_t)}{1+\eta\Lambda_t}\right) \\ &= |p_t(\theta_t^i - \theta_{t-1}^i)| \cdot \frac{1}{1+\eta}\left(1 + \frac{\eta(1+\Lambda_t)}{1+\eta\Lambda_t}\right), \quad \forall t > 0. \end{aligned}$$

The bubble increases the trading volume for all periods $t > 0$, when compared to the equivalent bubble-free equilibrium, and the increase factor is:

$$1 < \frac{1}{1+\eta}\left(1 + \frac{\eta(1+\Lambda_t)}{1+\eta\Lambda_t}\right) = \frac{1}{1+\eta}\left(2 - \frac{1-\eta}{1+\eta\Lambda_t}\right) \nearrow \frac{2}{1+\eta} \text{ (as } t \to \infty).$$

This analysis can be adapted to the case when the asset is in positive supply. Indeed, assume that $\theta_{-1}^1 = \theta_{-1}^2 = 1/2$, thus the asset is in unit supply. To account for the additional supply of goods in the economy due to dividends, and to preserve the structure of equilibrium, assume that agents' endowments of good are $\tilde{e}_t^i = e_t^i - d_t/2$. Thus the total endowment of goods of agent i if he sticks to his initial endowment of the asset is as before, since

$\tilde{e}_t^i + d_t/2 = e_t^i$, and equals y^H (y^L) if the agent is productive (unproductive). Moreover, assume that the debt constraints restrict only an agent's holdings in excess of initial endowment. Thus the budget and debt constraints are

$$c_t^i + p_t\tilde{\theta}_t^i = \tilde{e}_t^i + z_t^i n_t^i + (p_t + d_t)\tilde{\theta}_{t-1}^i,\ (p_t + d_t)(\tilde{\theta}_{t-1}^i - 1/2) \geq \bar{B}, \forall t \geq 0, i \in \{1, 2\}.$$

Letting $\theta_t^i := \tilde{\theta}_t^i - 1/2$, the budget and debt constraints become identical to zero supply case, and all the previous discussion goes through:

$$c_t^i + p_t\theta_t^i = e_t^i + z_t^i n_t^i + (p_t + d_t)\theta_{t-1}^i, (p_t + d_t)\theta_{t-1}^i \geq -\bar{B} \quad \forall t \geq 0, i \in \{1, 2\}.$$

Notice that a bubble ε in the asset produces here a contraction of agents' debt limits by $\varepsilon/2$, sterilizing the wealth effect of the bubble. Nevertheless, the bubble causes increases in trading volume due to portfolio rebalancing effects. Therefore, for assets in positive supply, bubbles can cause increases in trading volume even after their wealth effects are taken into account.

4 Conclusion

Rational bubbles can cause increases in trading volume, even after adjusting for their expansionary effect (on output, consumption and welfare). The increase in turnover is not due to speculation driven by differences in beliefs.

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